E C O N O M I C S B U L L E T I N

The Soft Budget Constraint Problem in a Dynamic Central Leadership Model

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Abstract

The authors deal with a certain type of timing problem with the central government's allocation of subsidies to local governments, called a ``soft budget constraint (SBC) problem." In fiscal science, it has been indicated that the interregional redistribution policy of the central government causes incentive problems such as excess expenditure or excess debt. However, as insignificant amount of the literature has applied a multi-period model to explain the SBC problem occuring despite the possibility of the existence of intertemporal distortion in an SBC situation. Therefore, the authors examine the problem in two-period models, which are the easiest multi-period models. The authors will demonstrate that the path of occurrence of the distortion caused by subsidization varies according to the timing of the subsidy offer.

This work was supported by Grant-in-Aid for Scientific Research (C) (18530226).
Citation: Takahashi, Hiromasa, Toru Takemoto, and Akihiro Suzuki, (2008) "The Soft Budget Constraint Problem in a Dynamic Central Leadership Model." *Economics Bulletin*, Vol. 8, No. 1 pp. 1-10
Submitted: January 11, 2008. **Accepted:** March 12, 2008.
URL: http://economicsbulletin.vanderbilt.edu/2008/volume8/EB-08H70010A.pdf

1. Introduction

In this paper, we deal with a certain type of timing problem with the central government's allocation of subsidies to local governments; this problem is referred to as the "soft budget constraint (SBC) problem." In fiscal science, it has been indicated that the interregional redistribution policy of the central government causes incentive problems such as excess expenditure or excess debt. However, an insignificant amount of literature has applied a multiperiod model to explain the occurrence of the SBC problem despite the possibility of the existence of intertemporal distortion in an SBC situation. We examine the problem in two-period models, which are the easiest multiperiod models. We will demonstrate that the path of occurrence of the distortion caused by subsidization varies according to the timing of the subsidy offer.

The origin of SBC is related to the analyses of distortions resulting from bailouts of loss-making state-owned enterprises in a socialized economy (see Kornai (1979, 1980)). Thereafter, SBC has been applied to the problem of subsidies provided to local governments by the central government. For example, Wildasin (1997) attempts an analysis of a situation wherein the central government provides ex-post subsidy relief to areas in which the consumption of public and private goods is in short supply as compared with that in other areas. It concludes that under such conditions, the budget constraints of local governments "soften" and the supply of public goods and the rate of tax are distorted.

In this paper, we suppose that there are two situations wherein local governments' decisions on the supply of local public goods and on the issue of local bonds and the central government's decisions on the delivery of subsidies are made at different timings. In one situation, the central government decides the subsidy as the first move of every period, and the local governments subsequently supply the local public goods; this is referred to as the "central leadership" (CL) model. In the other situation, the order of moves is inverted; this is known as the "decentralized leadership" (DL) model. Toy examples of these are described in Figure 1. In the subgame perfect equilibrium of the CL model, the central government plays "do not subsidize" and the local government plays "manage soundly." By contrast, the local government plays "manage loosely" and the central government plays "subsidize" in the equilibrium of the DL model. Therefore, the above examples show that a distortion occurs if the central government can, before choosing an action, observe the actions of the local government. This is the usual explanation of the SBC problem in the context of intergovernmental transfer, hereinafter we refer to the distortion as "basic SBC distortion." On the other hand, no inefficiency is found in the above CL model.

However, this result is not necessarily found only in a one-period model. If the relations between the central and local governments are maintained over the periods, the present decision making by local governments should have an influence on future decisions pertaining to central government subsidies. It can also be said that the present decision on the subsidy by the central government should have an influence on the future decision making of local governments. The point is that even if there is no distortion resulting from subsidy problems in the one-period model, distortion can be caused by simply shifting these problems to a multiperiod model. Furthermore, as compared to a one-period model generating distortion, there could be different mechanisms generating distortion in some multiperiod models. Goodspeed (2002) studies the SBC problem by using a two-period model with local bonds. However, the model of Goodspeed (2002) seems to be a nonstandard two-period model since two periods in the model are asymmetric. That is, for every government, the set of actions in the first period is not equal to that in the second period. Furthermore, we capture the SBC problem as a timing problem by dealing with models with a different order of moves, while the central government and local governments make decisions simultaneously in the period considered in Goodspeed (2002). Thus, the purpose of this paper is different from that of Goodspeed (2002).

This paper is organized as follows: We devote Section 2 to the setup and solution of the planning problem. In Sections 3 and 4, we introduce the two-period CL and DL models. In Section 5, we attempt a discussion on the subgame perfect equilibria of the previous sections. Section 6 concludes this paper.

2. Preliminaries

2.1 Objectives and Constraints

The economy contains two regions: region 1 and region 2. Further, the economy includes central and two local governments. A typical region or a local government is represented by i. Each region consists of a representative resident who lives for T periods. The resident in region i earns one unit of income in every period and his or her preference is represented by a utility function given by

$$U^i(\{c_t^i\}, \{g_t^i\}) = \sum_{t=1}^{T} \beta^{t-1}(\ln c_t^i + \ln g_t^i),$$

where T indicates the last period, assumed to be 1 or 2 in this paper; c_t^i indicates private goods consumption; and g_t^i indicates the local public goods[1] supply in period t. They are assumed to be nonnegative. The central government has social welfare

$$U^1(\{c_t^1\}, \{g_t^1\}) + U^2(\{c_t^2\}, \{g_t^2\}).$$

Both the local governments simultaneously decide the outstanding local bonds b_t^i and the local taxes q_t^i. We assume that b_0^i is given. In period T, the outstanding local bonds should be equal to $\bar{b}^i$. The central government decides the subsidies (s_t^1, s_t^2) to each resident to satisfy $s_t^1 + s_t^2 = 0$. After the decisions of the local governments and the central government, the consumption of private and public goods satisfies the following equations:

$$c_t^i = 1 - q_t^i + s_t^i, \quad (1)$$

$$g_t^i = q_t^i + b_t^i - (1+r)b_{t-1}^i, \quad (2)$$

where $r \in (0, 1)$ is the interest rate that is invariant with time and is set for all regions.

[1]Strictly speaking, the goods are not public goods. We could ascribe g_t^i to the properties of public goods; however, this would add unnecessary complexity to our assertion.

2.2 Planning Problem

The one-period planning problem is described in the following manner:

$$\max \sum_{i=1}^{2} \{\ln(1-q_1^i+s_1^i)+\ln[\,q_1^i+\bar{b}_1^i-(1+r)b_0^i]\}$$
$$\text{s.t.}\quad 1-q_1^i+s_1^i\geq 0,\; q_1^i+\bar{b}_1^i-(1+r)b_0^i\geq 0,$$
$$s_1^1+s_1^2=0,\; \text{and}\;\; b_0^1 \text{ and } b_0^2 \text{ are given.}$$

The two-period planning problem is described in the following manner:

$$\max \sum_{t=1}^{2} \beta^{t-1} \sum_{i=1}^{2} \{\ln(1-q_t^i+s_t^i)+\ln[\,q_t^i+b_t^i-(1+r)b_{t-1}^i]\}$$
$$\text{s.t.}\quad 1-q_t^i+s_t^i\geq 0,\; q_t^i+b_t^i-(1+r)b_{t-1}^i\geq 0,$$
$$s_t^1+s_t^2=0, b_2^i=\bar{b}_2^i,\; \text{and}\;\; b_0^1 \text{ and } b_0^2 \text{ are given.}$$

The consumption that maximizes social welfare satisfies $c_t^i = g_t^i$ for all i and t and $c_1^i=\beta(1+r)c_2^i$ for all i. The solutions to these problems are presented in Appendix A. We state that intratemporal consumption is efficient if $c_t^i = g_t^i$ and that intertemporal consumption is efficient if $c_1^i=\beta(1+r)c_2^i$ for all i.

3. DL Models

In this section, we present a one-period and a two-period DL model. The models in this paper are divided into two classes, the DL and CL models. In the DL models, both the local governments simultaneously move first, followed by the central government. In the CL models, the central government moves first, followed by the local governments. Akai and Sato (2005) show that there is distortion by SBC in the one-period DL model. On the other hand, Wildasin (1997) shows that the outcome is efficient in the one-period CL model. Differences in timing between these models cause differences in efficiency. The two-period version of the CL model reflects the same situation as the one-period DL model. The situation in this case means that the central government decides the subsidy after the local governments' actions. The situation may produce different results from the one-period CL model. Therefore, in the next section, we will consider a two-period CL model and show that inefficiencies are observed in the model.

3.1 One-Period DL Model

The players' moves in the extensive-form game of a one-period DL model are as follows:

(i) Both the local governments simultaneously decide the local taxes q_1^i.[2]

(ii) The central government decides the subsidies (s_1^1, s_1^2) to each resident such that they satisfy $s_1^1+s_1^2=0$.

The private goods consumption c_1^i and the local public goods supply g_1^i are realized, where c_1^i and g_1^i satisfy (1) and (2), respectively.

[2]By assumption, $b_1^i=\bar{b}^i$.

The strategy of local government i is q_1^i and that of the central government is $s_1^1(q_1^1, q_1^2)$. The payoff of local government i is

$$u^i(q_1^1, q_1^2, s_1^1) = \ln(1 - q_1^i + s_1^i) + \ln(q_1^i + \bar{b}^i - (1+r)b_0^i) \quad (s_1^2 = -s_1^1),$$

and the payoff function of the central government is

$$\begin{aligned} u^C(q_1^1, q_1^2, s_1^1) &\equiv u^1(q_1^1, q_1^2, s_1^1) + u^2(q_1^1, q_1^2, s_1^1), \\ &= \ln(1 - q_1^1 + s_1^1) + \ln(q_1^1 + \bar{b}^1 - (1+r)b_0^1) \\ &\quad + \ln(1 - q_1^2 - s_1^1) + \ln(q_1^2 + \bar{b}^2 - (1+r)b_0^2). \end{aligned}$$

The outcome is inefficient. See the appendices.

3.2 Two-Period DL Model

In this section, we describe the two-period DL model. This model is similar to a finitely repeated game except for the existence of a state variable. The "stage game" of this model is the one-period DL model described in the above section. Every player's payoff is the sum of the payoff obtained in the two-stage game. However, at the beginning of period 2, (b_1^1, b_1^2) is given as a state variable.

The strategy of local government i is $(q_1^i, b_1^i, q_2^i(q_1^1, b_1^1, q_1^2, b_1^2, s_1^1))$ and that of the central government is $(s_1^1(q_1^1, q_1^2, b_1^1, b_1^2), s_2^1(q_1^1, q_1^2, b_1^1, b_1^2, s_1^1, q_2^1, q_2^2))$. The payoff of local government i is

$$\begin{aligned} u^i(q_1^1, q_1^2, b_1^1, b_1^2, s_1^1, q_2^1, q_2^2, s_2^1) &= \ln(1 - q_1^i + s_1^i(\cdot)) + \ln(q_1^i + b_1^i - (1+r)b_0^i) \\ &\quad + \beta \ln(1 - q_2^i(\cdot) + s_2^i(\cdot)) \\ &\quad + \beta \ln(q_2^i(\cdot) + \bar{b}^i - (1+r)b_1^i), \end{aligned}$$

where $s_t^2 = -s_t^1$. The payoff function of the central government is

$$\begin{aligned} u^C(q_1^1, q_1^2, b_1^1, b_1^2, s_1^1, q_2^1, q_2^2, s_2^1) &= u^1(q_1^1, q_1^2, b_1^1, b_1^2, s_1^1, q_2^1, q_2^2, s_2^1) \\ &\quad + u^2(q_1^1, q_1^2, b_1^1, b_1^2, s_1^1, q_2^1, q_2^2, s_2^1). \end{aligned}$$

The subgame perfect equilibria of the game are calculated in Appendix B. The outcome is inefficient.

4. CL Models

4.1 One-Period CL Model

The players' moves in the extensive-form game of a one-period CL model are as follows:

(i) The central government chooses the subsidy to local government 1, s_1^1.

(ii) Both the local governments simultaneously choose their tax levels, q_1^1, q_1^2.

The private goods consumption c_1^i and the local public goods supply g_1^i are realized, where c_1^i and g_1^i satisfy (1) and (2), respectively.

The payoff functions are the same as those of the DL model. A strategy of the central government is s_1^1, that of local government 1 is $q_1^1(s_1^1)$, and that of local government 2 is $q_1^2(s_1^1)$.

4.2 Two-Period CL Model

Although the outcome in a one-period DL model is inefficient, that in a one-period CL model is efficient (see Appendices). Therefore, the timing of the governments' actions seems critically important. In fact, if the central government decides the subsidy after the local governments' actions, the local governments expect a central government bailout, and thus, they make inefficient decisions. The resource allocation in a one-period CL model is efficient since the local governments cannot expect a central government bailout in the model. However, if we consider a multiperiod CL model, the resource allocation may be inefficient since there are intertemporal interactions between central and local governments. Hence, in this section, we consider a two-period CL model.

This model is similar to a finitely repeated game except for the existence of state variable. The "stage game" of this model is the one-period DL model described in the above section. Every player's payoff is the sum of the payoff obtained in a two-stage game. However, at the beginning of period 2, (b_1^1, b_1^2) is given as the state variable.

The strategy of local government i is $(q_1^i(s_1^1), b_1^i(s_1^1), q_2^i(q_1^1, b_1^1, q_1^2, b_1^2, s_1^1, s_2^1))$ and that of the central government is $(s_1^1, s_2^1(q_1^1, q_1^2, b_1^1, b_1^2, s_1^1))$. The payoff of local government i is

$$\begin{aligned} u^i(s_1^1, q_1^1, q_1^2, b_1^1, b_1^2, s_2^1, q_2^1, q_2^2) &= \ln(1 - q_1^i(\cdot) + s_1^i) + \ln(q_1^i(\cdot) + b_1^i(\cdot) - (1+r)b_0^i) \\ &\quad + \beta \ln(1 - q_2^i(\cdot) + s_2^i(\cdot)) \\ &\quad + \beta \ln(q_2^i(\cdot) + \bar{b}^i - (1+r)b_1^i(\cdot)), \end{aligned}$$

where $s_t^2 = -s_t^1$. The payoff function of the central government is

$$\begin{aligned} u^C(s_1^1, q_1^1, q_1^2, b_1^1, b_1^2, s_2^1, q_2^1, q_2^2) &= u^1(s_1^1, q_1^1, q_1^2, b_1^1, b_1^2, s_2^1, q_2^1, q_2^2) \\ &\quad + u^2(s_1^1, q_1^1, q_1^2, b_1^1, b_1^2, s_2^1, q_2^1, q_2^2). \end{aligned}$$

The subgame perfect equilibrium of the game is calculated in Appendix C. The outcome is inefficient.

5. Discussion

5.1 Source of Distortion

One-Period Models The consumption of private goods should equal that of the local public goods for the utility to be maximized. In the one-period CL model, this condition is satisfied, namely, no inefficiency is found. On the other hand, in the one-period DL model, the consumption of private goods is only half as much as that of the local public goods under the subgame perfect equilibrium. The reason for this inefficiency is explained as follows: The central government allocates subsidies to the region imposing a higher local tax such that the private goods consumption in both the regions is identified. This gives local governments an incentive to impose heavier taxes. That is, the subsidy policy of the central government distorts the intratemporal resource allocation. We refer to the distortion in a period that is induced by the central government's actions in the same period as *basic SBC distortion*.

Two-Period DL Model Under the subgame perfect equilibrium, the intratemporal resource allocation is inefficient since the consumption of private goods is only half as much as that of the local public goods in each period under the subgame perfect equilibrium. The reason for the intratemporal inefficiency is the same as that in the one-period DL model, namely, basic SBC distortion.

The gross outstanding local bonds under the subgame perfect equilibrium are too high as compared with the optimal solution. This implies overconsumption in the first period and underconsumption in the second period. The reason for the distortion of the intertemporal resource allocation may be explained as follows: If a local government expands its outstanding local bonds in the first period, it raises the local taxes to redeem the local bonds or to pay the interest in the second period. This increase in the local taxes reduces the consumption of private goods. This reduction in private goods consumption is the cost of expanding the outstanding local bonds. However, the cost is reduced because the central government provides subsidies to the region where local taxes are high. In this manner, the subsidy policy of the central government in the second period gives the local governments an incentive to expand their outstanding local bonds in the first period. We refer to the distortion caused by the local governments' expectation of central-government-sponsored bailouts in the second period as *price effect distortion*. The reason for the distortion in the two-period DL model is the same as that in the infinitely iterated DL model examined in Takahashi, Takemoto, and Suzuki (2007).[3]

Two-Period CL Model Under the subgame perfect equilibrium, the intratemporal resource allocation is efficient in that the consumption of private goods equals that of the local public goods in each period.

The gross outstanding local bonds under the subgame perfect equilibrium are too high as compared with the optimal solution. This implies overconsumption in the first period and underconsumption in the second period. In this sense, the intertemporal resource allocation is inefficient under the subgame perfect equilibrium. Although no efficiency is found in the one-period CL model, inefficiency is observed in the two-period CL model. The reason for the distortion of the intertemporal resource allocation may be explained as follows: In the second period, the central government allocates subsidies to the region with higher outstanding local bonds such that the temporal utility in both the regions is the same. This gives the local governments an incentive to issue more debt in the first period. That is, the subsidy policy of the central government distorts the intertemporal resource allocation. We refer to the distortion that is induced by the central government's actions in the second period as *dynamic SBC distortion*.

The reason for the distortion in the two-period model is quite different from that in the infinitely iterated CL model examined in Takahashi et al. (2007). In the infinitely iterated CL model, an intertemporal distortion occurs in the outcome of a certain Markov perfect equilibrium (MPE), while an intratemporal distortion is not found. The distortion is referred to as direct overcompetitive distortion in Takahashi et al. (2007).[4]

The dynamic SBC distortion in the two-period model is quite different from the direct overcompetitive distortion in the following respect. In the infinitely iterated CL model, the local governments' strategies are not controllable by the central government's strate-

[3]Two-period models are naturally extended to t-period models for an arbitrary time period t, and the infinitely iterated DL and CL models can be interpreted as the limit of the t-period DL and CL models as $t \to \infty$.

[4]We omit a disquisition for saving space.

gies in the following sense: no matter how the central government varies its strategies, each local government has no incentive to deviate its strategies in constructing the MPE. In the infinitely iterated CL model, each local government has no interest in the delivery rule of the subsidies but is only concerned with the amount of the subsidies. On the other hand, in the two-period CL model, the local governments' strategies are controllable by the central government's strategies; in other words, the local governments change their strategies depending on the central government's strategy. In the two-period CL model, each local government is interested in not only the amount of the subsidies but also the delivery rule of the subsidies.

5.2 Social Welfare

$V^{CL}(B_0)$ and $V^{DL}(B_0)$ denote social welfares under the subgame perfect equilibrium in the two-period CL and DL models, respectively, and are as follows.

$$V^{CL}(B_0) = 2(1+\beta)\ln I(B_0) + 2\{\beta \ln \frac{\beta(1+r)}{2} - (1+\beta)\ln(2+\beta)\} - 2(1+\beta)\ln 2,$$

$$V^{DL}(B_0) = 2(1+\beta)\ln I(B_0) + 2\{\beta \ln \frac{2\beta(1+r)}{3} - (1+\beta)\ln(3+2\beta)\} - (1+\beta)\ln 2,$$

$$(\forall \beta \in (0,1)) \quad V^{CL} > V^{DL},$$

where $I(B_0) = 2 + \frac{2}{1+r} + \frac{\bar{b}^1+\bar{b}^2}{1+r} - (1+r)B_0$ and $B_0 = b_0^1 + b_0^2$.

In the one-period CL model, the outcome of the subgame perfect equilibrium is efficient, namely, the social welfare is maximized, whereas the outcome of the subgame perfect equilibrium in the one-period DL model is not.

On the other hand, social welfare in both the two-period CL and DL models, the social welfare is not maximized. The result in the one- and two-period models are consistent in that the CL model is more desirable than the DL model.

6. Conclusion

The results are summarized as follows:

- Inefficiency is observed in the two-period CL model, while it is not observed in the one-period CL model.

- In the two-period CL model, there exists no intratemporal distortion. However, an intertemporal distortion is observed in this model.

- An intertemporal distortion also occurs in the two-period DL model. The manner of appearence differs from that in the two-period CL model, although the reason for this are subsidies from the central government in both the models.

- The social welfare in the two-period CL model is higher than that in the two-period DL model.

Table 1 summarizes the distortion of resource allocation.

Appendices

Since the outcome of one-period models is same as the one in period 2 in two-period models, all appendices are devoted to describe the outcome of two-period models.

A. Solution of Planning Problem

In period 2, since the planner must decide $c_2^1 = g_2^1 = c_2^2 = g_2^2$, the result is as follows:

$$\begin{aligned}(s_2^i)^* &= \frac{1}{2}\left[(\bar{b}^j - (1+r)b_1^j) - (\bar{b}^i - (1+r)b_1^i)\right] (j \neq i),\\ (q_2^i)^* &= \frac{1}{2}\left[(1 + (s_2^i)^*) - (\bar{b}^i - (1+r)b_1^i)\right]\end{aligned}$$

$(\bar{b}^j - (1+r)b_1^j) - (\bar{b}^i - (1+r)b_1^i)$ means the difference of before-subsidy income between two regions. Therefore, the central government equalize after-subsidy income between two regions. Private goods and public goods consumption are

$$(c_2^i)^* = (g_2^i)^* = \frac{1}{4}\left[2 + \bar{b}^1 + \bar{b}^2 - (1+r)((b_1^1)^* + (b_1^2)^*)\right]$$

for all i.

Note that $c_1^1 = c_1^2 = g_1^1 = g_1^2$. We get $(c_1^1)^*$ by solving the following problem;

$$\begin{aligned}&\max\ 4 \ln c_1^1 + 4\beta \ln c_2^1\\ &\quad \text{s.t.}\quad 4\, c_1^1 + 4 \cdot \frac{c_2^1}{1+r} = I(B_0).\end{aligned}$$

$I(B_0) = 2 + \frac{2}{1+r} + \frac{\bar{b}^1+\bar{b}^2}{1+r} - (1+r)B_0$ and $B_0 = b_0^1 + b_0^2$ as defined in Section 5.

Therefore, private goods and public goods consumption in period 1 are

$$(c_1^1)^* = (c_1^2)^* = (g_1^1)^* = (g_1^2)^* = \frac{I(B_0)}{4(1+\beta)},$$

and $(c_2^1)^* = \beta(1+r)(c_1^1)^*$. The socially optimal outstanding local bonds in period 1 is

$$(b_1^1)^* + (b_1^2)^* = \frac{\beta}{1+\beta}\left[\frac{2 + \bar{b}^1 + \bar{b}^2}{\beta(1+r)} + (1+r)(b_0^1 + b_0^2) - 2\right].$$

B. Solution of DL Model

The central government assigns the subsidies in the second period 2 as follows:

$$\tilde{s}_2^1(q_2^1, q_2^2) = \frac{1}{2}(q_2^1 - q_2^2).$$

Local governments decide the local taxes and the outstanding local bonds as follows:

$$\begin{aligned}\tilde{q}_2^1(b_1^1, b_1^2) &= \frac{1}{3}[2 + \bar{b}^2 - 2\bar{b}^1 + (1+r)(2b_1^1 - b_1^2)],\\ \tilde{q}_2^2(b_1^1, b_1^2) &= \frac{1}{3}[2 + \bar{b}^1 - 2\bar{b}^2 + (1+r)(2b_1^2 - b_1^1)].\end{aligned}$$

In period 2, private goods consumption is

$$\tilde{c}_2^1 = \tilde{c}_2^2 = \frac{1}{6}[2 + \bar{b}^1 + \bar{b}^2 - (1+r)(\tilde{b}_1^1 + \tilde{b}_1^2)],$$

and public goods consumption is

$$\tilde{g}_2^1 = \tilde{g}_2^2 = \frac{1}{3}[2 + \bar{b}^1 + \bar{b}^2 - (1+r)(\tilde{b}_1^1 + \tilde{b}_1^2)].$$

Decision of local governments in period 2 is invariant by the subsidies from the central government in period 1. Therefore, the action of the central government in period 2 is

$$\tilde{s}_1^1(q_1^1, q_1^2) = \frac{1}{2}(q_1^1 - q_1^2).$$

The central government decides subsidies to help the resident more paying local tax. That is, the central government equalizes private goods consumption between two regions.

The F.O.C. for q_1^i of local government i is

$$q_1^i = \frac{1}{3}(2 + b_1^1 + b_1^2 - (1+r)(b_0^1 + b_0^2)) - (b_1^i - (1+r)b_0^i),$$

and the F.O.C. for b_1^i of local government i is

$$q_1^i + b_1^i - (1+r)\bar{b}^i = \frac{2 + \bar{b}^1 + \bar{b}^2 - (1+r)(b_1^1 + b_1^2)}{2\beta(1+r)}.$$

Since these four equations are linearly dependent, if $(\tilde{q}_1^1, \tilde{q}_1^2, \tilde{b}_1^1, \tilde{b}_1^2)$ satisfies the following equations,

$$\begin{aligned}
\tilde{q}_1^1 + \tilde{q}_1^2 &= 2 - \frac{1}{3+2\beta} I(B_0), \\
\tilde{b}_1^1 + \tilde{b}_1^2 &= \frac{3}{3+2\beta} I(B_0) - 2 + (1+r)B_0, \text{ and} \\
\tilde{g}_1^i &= \frac{1}{3+2\beta} I(B_0),
\end{aligned}$$

$(\tilde{q}_1^1, \tilde{q}_1^2, \tilde{b}_1^1, \tilde{b}_1^2)$ is a part of an equilibrium. In any equilibrium, private and local public goods consumption is as follows:

$$2\tilde{c}_1^1 = 2\tilde{c}_1^2 = \tilde{g}_1^1 = \tilde{g}_1^2 = \frac{I(B_0)}{3+2\beta}.$$

C. Solution of CL Model

Local governments' actions in period 2 are

$$\begin{aligned}
\hat{q}_2^1(s_2^1, b_1^1) &= \frac{1}{2}(1 + s_2^1 - \bar{b}^1 + (1+r)b_1^1), \\
\hat{q}_2^2(s_2^1, b_1^2) &= \frac{1}{2}(1 - s_2^1 - \bar{b}^2 + (1+r)b_1^2),
\end{aligned}$$

and the central government's action is

$$\hat{s}_2^1(b_1^1, b_1^2) = \frac{1}{2}(\bar{b}^2 - \bar{b}^1 - (1+r)(b_1^2 - b_1^1)).$$

Equilibrium bonds of local government i in period 1 are

$$\hat{b}_1^i = -(1 + s_1^i) + (1+r)b_0^i + \frac{1}{2+\beta}I(B_0).$$

The local tax of local government i in period 1 is

$$\hat{q}_1^i = (1 + s_1^i) - \frac{1}{2} \cdot \frac{1}{2+\beta}I(B_0).$$

Therefore, the central government takes any s_1^1 in equilibrium since c_1^i and g_1^i are independent of s_1^1. In any equilibrium, the consumption of private and local public good is

$$\hat{c}_1^i = \hat{g}_1^i = \frac{1}{2(2+\beta)}I(B_0)$$

Since

$$\hat{c}_2^i = \frac{1}{2}\beta(1+r)\hat{c}_1^i,$$

the outcome of the equilibrium in the two-period CL model is inefficient. While there is no distortion of resource allocation intratemporally, there are intertemporal distortions.

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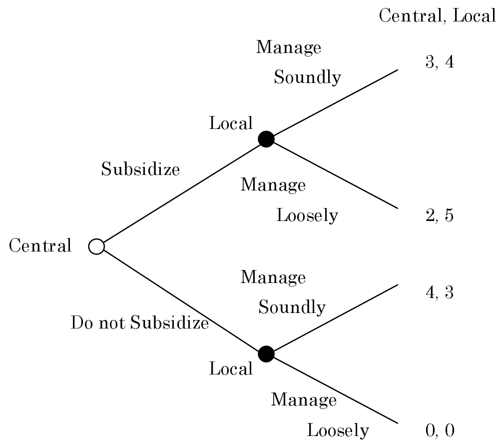
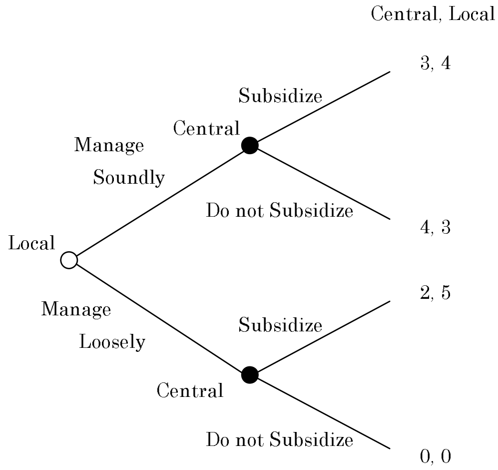
Figure 1: Toy Examples



(a) One-Shot CL Model



(b) One-Shot DL Model

Table 1: Comparison of Distortion

Model	Intratemporal Distortion	Intertemporal Distortion
One-Period CL	Nonexistence	—
One-Period DL	Basic SBC Distortion	—
Two-Period CL	Nonexistence	Dynamic SBC Distortion
Two-Period DL	Basic SBC Distortion	Price Effect Distortion