E C O N O M I C S B U L L E T I N

The estimation of the growth and redistribution components of changes in poverty: a reassessment

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Abstract

What are the respective contributions of growth and inequality changes to observed poverty variations? Many studies have attempted to provide some empirical evidence to answer this question using case studies with decompositions of observed poverty spells. Most of them rely on two decomposition frameworks suggested by Datt Ravallion (1992) on the one hand, and Shorrocks (1999) and Kakwani (2000) on the other hand. However, despite their properties, these techniques are not appropriate for such an accounting exercise. Here, following Muller (2006), we propose an alternative decomposition procedure that is consistent with definitions of growth and inequality effects stemming from time-integral calculus. Contrary to the aforementioned methods, the proposed technique simultaneously fits the observed pattern of income distributions changes and does not produce large residual components.

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1 INTRODUCTION

What are the respective contributions of growth and inequality changes to observed poverty variations? Inspired by the theoretical debates concerning the validity of "trickle-down" mechanisms and the necessity to achieve "pro-poor" growth, many studies have tried to provide some empirical evidence to this question. Many of them are case studies with decompositions of observed poverty spells (see Bhanumurthy and Mitra, 2004, Wan and Zhang, 2006, Baye, 2006, Dhongde, 2007, for recent illustrations). Generally, these studies rely on two decompositions frameworks suggested by Datt and Ravallion (1992) on the one hand, and Shorrocks (1999) and Kakwani (2000) on the other hand. However, we argue that these techniques are not appropriate for such an accounting exercise in spite of their attractive properties. Here, following Muller (2006), we propose an alternative decomposition procedure that is consistent with definitions of growth and inequality effects stemming from time-integral calculus. Contrary to aforementioned methods, the proposed technique simultaneously fits the observed pattern of income distributions changes and does not yield large residual components.

Section 2 reviews traditional decompositions techniques in single- and multi-period contexts while section 3 is dedicated to the presentation of the new decomposition method. Section 4 concludes with further comments.

2 EXISTING DECOMPOSITIONS

Let Θ be any absolute poverty measure that complies with traditional core axioms of poverty measurement (see Zheng, 1997, for a review).[1] A common practice in the poverty related literature is to characterize an income distribution by its mean value and its relative distribution. The measure Θ can thus be expressed as a function of the poverty line z, the mean income μ, a set $\boldsymbol{\pi}$ of r parameters that fully describes the relative income distribution, and a vector $\boldsymbol{\alpha}$ that accounts for ethical preferences of the social evaluator.[2] Hence, $\Theta = \Theta(z, \mu, \boldsymbol{\pi}, \boldsymbol{\alpha})$. We suppose that the poverty line is time-invariant, an assumption that entails to consider real incomes. In the same manner, the vector $\boldsymbol{\alpha}$ is assumed constant since there is little reason to compare the level of poverty between two dates on the basis of different ethical preferences. As a consequence, the notation of Θ can be simplified into $\Theta(\mu, \boldsymbol{\pi})$.

The objective of the decompositions considered in the present paper, is to assess the

[1] To the list of core axioms defined by Zheng (1997), some invariance axiom should also be added so as to get an oprational set of axioms for poverty measurement. Usually, the chosen poverty measures are based on the scale invariance axiom, but our developments are also valid for measures based on rival invariance axioms (see Bresson and Labar, 2007, for a recent review of invariance axioms).

[2] Generally, $\boldsymbol{\pi}$ is defined as a vector of parameters that fully describes the Lorenz curve. This interpretation is normativelly non-neutral since it entails focusing on the sole poverty measures that comply with the scale invariance axiom. As this choice can be debated (see Amiel and Cowell, 1992, for empirical evidence), we prefer a more general and neutral interpretation.

respective contribution of mean income growth and inequality changes in observed poverty variations. Let $\mathscr{C}$ denote that theoretical contribution of growth and $\mathscr{I}$ the one attributed to relative distribution changes. As μ and $\boldsymbol{\pi}$ are the sole variable determinants of Θ, our goal is to obtain the following decomposition between dates t and $t+k$:

$$\Theta(\mu_{t+1}, \boldsymbol{\pi}_{t+k}) - \Theta(\mu_t, \boldsymbol{\pi}_t) = \Delta_{t,t+k}\Theta(\mu, \boldsymbol{\pi}) = \mathscr{C}_{t,t+k} + \mathscr{I}_{t,t+k}. \tag{2.1}$$

2.1 The single period case

In spite of the apparent simplicity of the procedure, the computation of $\mathscr{C}$ and $\mathscr{I}$ is not straightforward because of the intrinsic non-separability of Θ ($\partial^2\Theta/\partial\mu\partial\pi_i \neq 0 \forall i \in \{1\dots r\}$).[3] This explains the existence of rival procedures in the literature for the calculation of these growth and inequality effects. Here we focus on the decomposition procedures proposed by Datt and Ravallion (1992) at one hand (thereafter called DR decomposition), and Shorrocks (1999) and Kakwani (2000) at the other hand (thereafter noted KS decomposition).[4] In the former case, the suggested decomposition is:

$$\Delta_{t,t+1}\Theta(\mu, \boldsymbol{\pi}) = \underbrace{\Theta(\mu_{t+1}, \boldsymbol{\pi}_t) - \Theta(\mu_t, \boldsymbol{\pi}_t)}_{\mathscr{C}_{t,t+1} := C_{t,t+1}} + \underbrace{\Theta(\mu_t, \boldsymbol{\pi}_{t+1}) - \Theta(\mu_t, \boldsymbol{\pi}_t)}_{\mathscr{I}_{t,t+1} := I_{t,t+1}} + R_{t,t+1}, \tag{2.2}$$

with R being a residual term. The DR procedure is very appealing since C and I exhibit very intuitive meanings: the growth (inequality) effect C (I) corresponds to the variation of Θ that would have occured for the observed rate of growth (inequality change) if the realtive income distribution (the mean income) was held fixed. This means that the proposed formulas estimates the respective impacts of mean income and inequality changes since they are computed through a comparison with counterfactual situations.

The residual term R appears when both μ and $\boldsymbol{\pi}$ varies, that is in nearly all cases. The authors interpret it as the difference between the inequality effect computed with respect to mean income in $t+1$ and the one obtained with respect to mean income at date t, that is $I_{t+1,t} - I_{t,t+1}$. However, they notice that it can equivalently be seen as the difference $C_{t+1,t} - C_{t,t+1}$. In other words, it represents the interactive effect of the simultaneous variations of μ and $\boldsymbol{\pi}$ on poverty. In practice, the size of this interactive term may be non-negligible.[5] This is a rather puzzling result since it represents a clear failure in accounting for the whole respective contributions of growth and inequality changes in observed poverty variations. To get a decomposition which respects relation (2.1), it is then necessary to share

[3] A noticeable exception is the class of poverty measures defined by Tsui (1996).

[4] In order to save space, the methods suggested by Jain and Tendulkar (1990) and Kakwani and Subbarao (1990) have been skipped. The main drawback of these procedures is that they cannot be used to compare the relative contribution of growth and inequality changes in terms of poverty alleviation since they are not computed on the same basis.

[5] Considering the empirical illustration conducted by the authors, the residual term account for about fifty percents of total poverty variations in some cases.

the residual term R between the growth and inequality effects.

The solution suggested by Shorrocks (1999) relies on the use of the Shapley (1953) value developed in game theory so as to define an efficient rule for the sharing of gains among players. The intuition behind this rule is that every participant should gain the mean value of the marginal contributions that he adds in every possible coallitions with the other players. A formal general presentation of the Shapley value is given in appendix A. Its application to poverty variation yields the following decompostion:

$$\Delta_{t,t+1}\Theta(\mu,\boldsymbol{\pi}) = \overbrace{\frac{\big(\Theta(\mu_{t+1},\boldsymbol{\pi}_t)-\Theta(\mu_t,\boldsymbol{\pi}_t)\big)+\big(\Theta(\mu_{t+1},\boldsymbol{\pi}_{t+1})-\Theta(\mu_t,\boldsymbol{\pi}_{t+1})\big)}{2}}^{\mathscr{C}_{t,t+1}:=C'_{t,t+1}} + \underbrace{\frac{\big(\Theta(\mu_t,\boldsymbol{\pi}_{t+1})-\Theta(\mu_t,\boldsymbol{\pi}_t)\big)+\big(\Theta(\mu_{t+1},\boldsymbol{\pi}_{t+1})-\Theta(\mu_{t+1},\boldsymbol{\pi}_t)\big)}{2}}_{\mathscr{I}_{t,t+1}:=I'_{t,t+1}}. \tag{2.3}$$

The decomposition has also been proposed by Kakwani (2000) who followed an axiomatic approach to derive equation (2.3). The author argued that the three following conditions should be met for a decomposition procedure to be valid: *i)* when the growth (inequality) effect is zero, then the change in poverty must be entirely due to change in income inequality (mean income), *ii)* if both growth and inequality effects are negative (positive), then poverty should decline (increase), and *iii)* the growth (inequality) effect from the initial to the final date must be the opposite of the growth (inequality) from the final to the initial date. It can easily be shown that $C'_{t,t+1} = C_{t,t+1} + \frac{R_{t,t+1}}{2}$ and $I'_{t,t+1} = I + \frac{R_{t,t+1}}{2}$.[6]

2.2 The multiple periods case

If the decomposition is applied in a multiple periods context, both Datt and Ravallion (1992) and Kakwani (2000) notice that the formulas presented for the single period context are not valid any more since they do not comply with the sub-period additivity principle. Suppose we are analyzing the evolution of poverty over a period 0 to T. Let t, u and v be three dates such that $0 \leqslant t \leqslant u \leqslant v \leqslant T$, $\{t,u,v,T\} \in \mathbb{N}$. Sub-period additivity means that the growth (ineqality) effect between t and v should be the sum of the estimated growth effects for the sub-periods t to u and u to v, *i.e.* in formal terms:[7]

$$\mathscr{C}_{t,v} = \mathscr{C}_{t,u} + \mathscr{C}_{u,v} \quad \text{and} \quad \mathscr{I}_{t,v} = \mathscr{I}_{t,u} + \mathscr{I}_{u,v}. \tag{2.4}$$

However, this property is not satisfied by any of the decomposition procedures presented in the last section. Therefore, to deal with this issue, one has to choose between changing the formula corresponding to the multiperiod effects, modifying the equations

[6] This solution has already been considered by Datt and Ravallion (1992) but judged arbitrary.
[7] In Datt and Ravallion (1992), the sub-period additivity condition is also applied to the residual term.

related to the sub-period effects, or revising both expressions .

In Datt and Ravallion (1992), the second solution is adopted: sub-period additivity is satisfied if sub-periods effects are computed with respect to a unique reference income distribution. As the initial distribution of the whole period (date 0) can be considered as a natural choice, equation (2.2) is modified in the following way:

$$\Delta_{t,u}\Theta(\mu,\boldsymbol{\pi}) = \underbrace{\Theta\left(\mu_0+\Delta_{t,u}\mu,\boldsymbol{\pi}_0\right)-\Theta(\mu_0,\boldsymbol{\pi}_0)}_{\mathscr{C}_{t,u}:=C^0_{t,u}} + \underbrace{\Theta\left(\mu_0,\boldsymbol{\pi}_0+\Delta_{t,u}\boldsymbol{\pi}\right)-\Theta(\mu_0,\boldsymbol{\pi}_0)}_{\mathscr{I}_{t,u}:=I^0_{t,u}} + R_{t,u}, \qquad (2.5)$$

where the superscript indicates that year 0 has been chosen for the reference distribution.

The multi-period decompostion proposed by Kakwani (2000) consists in redefining both sub-period and whole-period effects. It relies on a version of the sub-period additivity principle that is more restrictive than the one defined by Datt and Ravallion (1992) since conditions (2.4) have to hold $\forall u \in \{0,1\dots T\}$. The introduction of this condition in his axiomatic framework yields the following the decomposition:

$$\Delta_{t,u}\Theta(\mu,\boldsymbol{\pi}) = \underbrace{\frac{1}{T}\sum_{j=0}^{T}\left(C'_{t,j}+C'_{j,u}\right)}_{\mathscr{C}_{t,u}:=\tilde{C}^{0,T}_{t,u}} + \underbrace{\frac{1}{T}\sum_{j=0}^{T}\left(I'_{t,j}+I'_{j,u}\right)}_{\mathscr{I}_{t,u}:=\tilde{I}^{0,T}_{t,u}}, \qquad (2.6)$$

where the superscript $0,T$ corresponds to the reference period on which are based the estimations.

Although the multi-period context has not been investigated by Shorrocks (1999), it can easily be seen that decomposition (2.6) is a departure from the application of the Shapley value. Indeed, a literally use of this tool would require to dismiss the sequence of observed changes in the income distribution. As the Shapley value is computed using the different possible sequence of players participations, marginal contributions of growth and inequality changes would be estimated using every arrangements of the different $\Delta_{t,t+1}\mu$ and $\Delta_{t,t+1}\boldsymbol{\pi}$ that are observed over the whole period. For instance, in a two period context (dates 0, 1 and 2), using the Shapley decomposition entails considering the 24 arrangements of $\Delta_{0,1}\mu$, $\Delta_{0,1}\boldsymbol{\pi}$, $\Delta_{1,2}\mu$ and $\Delta_{1,2}\boldsymbol{\pi}$. The corresponding growth effect C^S for the second period would then be:

$$\begin{aligned}C^S_{1,2} = {} & \frac{1}{4}\big(\Theta(\mu_0+\Delta_{1,2}\mu,\boldsymbol{\pi}_0)-\Theta(\mu_0,\boldsymbol{\pi}_0)\big)+\frac{1}{12}\big(\Theta(\mu_2,\boldsymbol{\pi}_0)-\Theta(\mu_1,\boldsymbol{\pi}_0)\big)+\frac{1}{12}\big(\Theta(\mu_0 \\ & +\frac{1}{12}\big(\Theta(\mu_0+\Delta_{1,2}\mu,\boldsymbol{\pi}_0+\Delta_{1,2}\boldsymbol{\pi})-\Theta(\mu_0,\boldsymbol{\pi}_0+\Delta_{1,2}\boldsymbol{\pi})\big)+\frac{1}{12}\big(\Theta(\mu_2,\boldsymbol{\pi}_1)-\Theta(\mu_1,\boldsymbol{\pi}_1)\big) \\ & +\frac{1}{12}\big(\Theta(\mu_2,\boldsymbol{\pi}_0+\Delta_{1,2}\boldsymbol{\pi})-\Theta(\mu_1,\boldsymbol{\pi}_0+\Delta_{1,2}\boldsymbol{\pi})\big)+\frac{1}{12}\big(\Theta(\mu_0+\Delta_{1,2}\mu,\boldsymbol{\pi}_2)-\Theta(\mu_0,\boldsymbol{\pi}_2)\big) \\ & +\frac{1}{4}\big(\Theta(\mu_2,\boldsymbol{\pi}_2)-\Theta(\mu_1,\boldsymbol{\pi}_2)\big)+\Delta_{1,2}\mu,\boldsymbol{\pi}_1)-\Theta(\mu_0,\boldsymbol{\pi}_1)\big).\end{aligned} \qquad (2.7)$$

which clearly differs from the effect stemming from equation (2.6).

2.3 DRAWBACKS

As noted earlier, the DR decomposition framework has been widely criticized due to the presence of a non-negligible residual term. Indeed, it represents a failure in accounting for the whole respective contributions of growth and inequality changes in observed poverty variations as the observer is left with a black-box that cannot be easily interpreted. The crucial point with this technique is that it does not measure what it is designed for since it is based on a confusion between "contributions" and "impacts". Indeed, DR effects can be interpreted as the impacts of growth and inequality changes on the level of poverty since they corresponds to the variation of poverty that would have occured for a given factor's variation *other things being equal.* A consequence is that DR growth and inequality effects are perfectly orthogonal as they are estimated using counterfactual situations. However, $\mathscr{C}$ and $\mathscr{I}$ are simultaneously determined and our feeling is that the estimation of contributions should not skip this association. This remark does not mean that the DR decomposition is false but that it does not suit the estimation of the global contributions of growth and inequality changes.

An other issue is that the solution suggested in the multiple period context is puzzling since the DR effects do not exhibit such an appealing interpretation as for the single period context. For instance, we could question the interest of knowing $C^0_{t,t+1}$ for $t \neq 0$ since it corresponds to the change in inequality that would have occured if income inequality were constant *for the income distribution at the reference year.* Thus it is very poorly related to observed poverty in dates t and $t+1$.

The KR decomposition framework is not free from all criticism. Recently, Muller (2006) argued that the single-period KS decomposition procedure is not satisfying and that its theoretical background is confusing (statistical phenomenons are not players). In particular, he considers that the evaluation of any decomposition technique should not be based on the capacity to provide a null residual or to comply with any practical mathematical property. In this spirit, we emphasize that the desirability of the third axiom proposed by Kakwani (2000) should be questioned. Yet, the aim of poverty variation decompositions is to assess the contribution of factors that drive the evolution of poverty through time. Fundamentally, since time development is a one-way road, it does not matter whether the effects estimated when considering a move from the income distribution $t+1$ to the one in t are equal (in absolute value) to those that effectively corresponds to observed evolutions from t to $t+1$.

One can also notice that the multi-period decomposition defined in equation (2.6) implies that the estimated growth and inequality effects are path-dependant. From this equation, it can be seen that any effect corresponding to a given sub-period depends on income distribution that are considered before, during and after the sub-period of interest. For instance, in the case of a four-period analysis (with years $0,1,2,3,4$), the estimation of $\tilde{C}^{0,4}_{1,3}$ would not only be driven by the income distributions in years 1 and 3, but also with those observed for the years 0, 2 and 4. While the influence of the income distribution in year 2 may be reasonnably be accepted, it seems more difficult with the initial and final distributions.

3 An integral-based decomposition procedure

Muller's (2006) most forceful argument against the aforementioned decomposition techniques is that they are not consistent with the theory of integration. Indeed, the observed poverty variation between t and $t+1$ can be rewritten in the following manner:

$$\Delta_{t,t+1}\Theta(\mu,\boldsymbol{\pi}) = \int_t^{t+1} \frac{d\Theta(\mu_q,\boldsymbol{\pi}_q)}{dq}\,dq, \tag{3.1}$$

where q is time. Equation (3.1) can easily be linked to the issue of poverty variations decompositions into growth and inequality components. Using a first-order Taylor expansion, we obtain:

$$\Delta_{t,t+1}\Theta(\mu,\boldsymbol{\pi}) \simeq \underbrace{\int_t^{t+1} \frac{\partial\Theta(\mu_q,\boldsymbol{\pi}_q)}{\partial\mu}\frac{\partial\mu}{\partial q}\,dq}_{\mathscr{C}_{t,t+1}} + \underbrace{\sum_{i=1}^{r}\int_t^{t+1} \frac{\partial\Theta(\mu_q,\boldsymbol{\pi}_q)}{\partial\pi_i}\frac{\partial\pi_i}{\partial q}\,dq}_{\mathscr{I}_{t,t+1}}. \tag{3.2}$$

Comparing equations (3.2) and (2.2) is informative about the nature of the DR decomposition. This latter also corresponds to a first-order Taylor expansion of $\Delta_{t,t+1}\Theta(\mu,\boldsymbol{\pi})$, but the first-order derivatives are substituted by linear interpolations all over the time interval. An alternative expression of equation (2.2) is then:

$$\begin{aligned}\Delta_{t,t+1}\Theta(\mu,\boldsymbol{\pi}) = {} & \overbrace{\frac{\Theta(\mu_2,\boldsymbol{\pi}_1)-\Theta(\mu_1,\boldsymbol{\pi}_1)}{\mu_2-\mu_1}(\mu_2-\mu_1)}^{C_{t,t+1}} \\ & \underbrace{+\sum_{i=1}^{r}\frac{\Theta(\mu_1,\pi_{i,2},\boldsymbol{\pi}_1^{-i})-\Theta(\mu_1,\boldsymbol{\pi}_1)}{\pi_{i,2}-\pi_{i,1}}(\pi_{i,2}-\pi_{i,1})}_{I_{t,t+1}} + R_{t,t+1}.\end{aligned} \tag{3.3}$$

where $\boldsymbol{\pi}^{-i}$ is the vector $\boldsymbol{\pi}$ minus its i-th component. The presence of the residual term in (3.3) can be explained by the non-linear relation between Θ and its different arguments. As observed variations of μ and $\boldsymbol{\pi}$ are generally not marginal, this approximation is unlikely to yield satisfying approximations of equation (3.2), except for small variations of Θ's determinants. If anyone intends to realize a decomposition that is based on equation (3.2), it is thus necessary to have an estimation of the first-order derivatives of Θ with respect to μ and $\boldsymbol{\pi}$ all over the intervals defined by their initial and final values. A solution is the use of local first-order Taylor expansion for marginal variations of each factor. Assuming that the evolution of these parameters are linear between t and $t+1$ (*i.e.* $\partial\mu/\partial q$ and $\partial\pi_i/\partial q$ are constant over the time interval $[t, t+1]$), we can divide $\Delta_{t,t+1}\mu$ and $\Delta_{t,t+1}\boldsymbol{\pi}$ into s small identical variations. Then, for each variation $j = 1\dots s$, first order derivatives of the poverty measure can be approximated in the following manner:

$$\frac{\partial}{\partial\mu}\Theta\left(\mu_{t+\frac{j-1}{s}}, \boldsymbol{\pi}_{t+\frac{j-1}{s}}\right) \simeq \frac{\Theta\left(\mu_{t+\frac{j}{s}}, \boldsymbol{\pi}_{t+\frac{j-1}{s}}\right) - \Theta\left(\mu_{t+\frac{j-1}{s}}, \boldsymbol{\pi}_{t+\frac{j-1}{s}}\right)}{\mu_{t+\frac{j}{s}} - \mu_{t+\frac{j-1}{s}}}, \tag{3.4}$$

$$\frac{\partial}{\partial\pi_i}\Theta\left(\mu_{t+\frac{j-1}{s}}, \boldsymbol{\pi}_{t+\frac{j-1}{s}}\right) \simeq \frac{\Theta\left(\mu_{t+\frac{j-1}{s}}, \pi_{i,t+\frac{j}{s}}, \boldsymbol{\pi}^{-i}_{t+\frac{j-1}{s}}\right) - \Theta\left(\mu_{t+\frac{j-1}{s}}, \boldsymbol{\pi}_{t+\frac{j-1}{s}}\right)}{\pi_{i,t+\frac{j}{s}} - \pi_{i,t+\frac{j-1}{s}}}. \tag{3.5}$$

Plugging these approximations into equation (3.2) yields the following decomposition:

$$\begin{aligned}\Delta_{t,t+1}\Theta(\mu,\boldsymbol{\pi}) \simeq\ & \overbrace{\sum_{j=1}^{s}\Theta\left(\mu_{t+\frac{j}{s}}, \boldsymbol{\pi}_{t+\frac{j-1}{s}}\right) - \Theta\left(\mu_{t+\frac{j-1}{s}}, \boldsymbol{\pi}_{t+\frac{j-1}{s}}\right)}^{\mathscr{C}_{t,t+1}:=\hat{C}_{t,t+1}} \\ & \underbrace{+\sum_{j=1}^{s}\sum_{i=1}^{r}\Theta\left(\mu_{t+\frac{j-1}{s}}, \pi_{i,t+\frac{j}{s}}, \boldsymbol{\pi}^{-i}t+\frac{j-1}{s}\right) - \Theta\left(\mu_{t+\frac{j-1}{s}}, \boldsymbol{\pi}_{t+\frac{j-1}{s}}\right)}_{\mathscr{I}_{t,t+1}:=\hat{I}_{t,t+1}}.\end{aligned} \tag{3.6}$$

which tends to equal equation (3.2) as $s \to +\infty$. Thus, the suggested procedure consists in decomposing observed variations of μ and $\boldsymbol{\pi}$ into s small equal variations and then, for each micro-variation, to compute the growth and inequality effects stemming from the DR decomposition. Summing up these s growth (inequality) micro-effects gives the desired global growth (inequality) effect. The practical implementation of this technique is reported in appendix B.

When considering poverty variations between dates t and $t+k$, the information related to mean income and inequality changes is more complete since the income distributions in $t+1,\ t+2\dots t+k-1$ can be observed. Consequently, the assumption that the evolution of μ and $\boldsymbol{\pi}$ was linear during the whole period is not necessary anymore and additional information should be exploited for the computation of the multi-period effects. However, the hypothesis of linear evolutions of growth and inequality during each subperiod has to be maintained. The multi-period decomposition can be then defined as:

$$\Delta_{t,t+k}\Theta(\mu,\boldsymbol{\pi}) \simeq \underbrace{\sum_{j=1}^{k}\hat{C}_{t+j-1,t+j}}_{\hat{C}^{\Sigma}_{t,t+k}} + \underbrace{\sum_{j=0}^{k}\hat{I}_{t+j-1,t+j}}_{\hat{I}^{\Sigma}_{t,t+k}}. \tag{3.7}$$

The chosen strategy is thus at odd with the one adopted by Datt and Ravallion (1992) and Kakwani (2000) since it consists in adapting the single-period decomposition for the multiple periods context. This extension to the multi-period context is elegant since it does not impose endorsing mathematical contorsions for the respect of sub-period additivity, and can be easily justified from an informational point of view. Finally, it is worth noting that the proposed decomposition is path-dependant. Nonetheless, contrary to Kakwani's (2000) multi-period decomposition, estimated effects do not depend of changes that have occured during the periods 0 to t and $t+k$ to T. The results obtained over any subperiod are thus consistent with those get over the whole period.

4 CONCLUDING REMARKS

In the following paper, we proposed a new decomposition framework to account for the respective contributions of growth and inequality changes to poverty variations. Based on first-order Taylor expansions and time-integral definitions of the growth and inequality effects, the proposed procedure defines a consistant rule for sharing the residual term produced by the Datt and Ravallion's (1992) decomposition. Our solution also contrasts with other decomposition techniques in the way multiple period decompositions are handled.

Despite the drawbacks noted in section 2.3, it is worth noting that the DR and KS decomposition frameworks should be relegated to historical surveys of poverty analysis. For instance, as explained earlier, the single-period DR decomposition is the right choice if one intends to estimate the respective impacts (*i.e. ceteris paribus effects*) of growth and inequality changes on the level of poverty. Regarding the Shapley value, Shorrocks's (1999) intention was to provide a framework that could be used for both static and dynamic general decompositions of poverty measures. As our remarks are only related to a specific dynamic decomposition, the Shapley value may still remain a useful tool for static analysis (see Sastre and Trannoy, 2002, in the context of inequality measure decompositions).

Finally, along these lines, we just focused on decompositions of poverty variations into growth and inequality changes components. However, many other decompositions can be handled. For instance, Son (2003) suggests a four-element decomposition such that the inequality effect is itself decomposed into *i)* the contribution of differences in growth rates between the groups, *ii)* the effect of the change in inequality within the different groups, and *iii)* the outcome of changes in the population shares of the various groups. The decomposition can also be extended if current income are considered instead of real income for poverty measurement. This slight modification entails the addition of a purchasing power effect to the growth and inequality effects. The extension of the decomposition method presented here to such cases is beyond the scope of this paper, but should deserve further scrutiny.

Appendix

A The Shapley value

Consider a set K of $m \in \mathbb{N}^*$ players which mutual efforts produced a gain $G \in \mathbb{R}$. Let a coallition S be a subset from K of s players. If $g(S)$ is the gain (or the loss) that can get the coallition S without negociating with the remaining players, the marginal contribution of player k when joining this coallition will be $g(S \cup k) - g(S)$. The solution suggested by Shapley (1953) is that each player k should be rewarded with its global contribution $V(k)$, that is the arithmetic mean of each one of its possible marginal contributions wheighted by the probability of observing each coallition. In formal terms, the "just" reward is:

$$V(k) = \sum_{s=0}^{m-1} \sum_{S \subseteq K-k} \frac{s!(m-s-1)!}{m!} \big(g(S \cup k) - g(S)\big). \tag{A.1}$$

Whether the marginal contributions g depends or not on the other players' actions, an interesting property of the Shapley value is that:

$$\sum_{k=1}^{m} V(k) = G. \tag{A.2}$$

B The integral-based decomposition in practice

Let $\boldsymbol{X}$ and $\boldsymbol{Y}$ be the income vectors that respectively corresponds to the initial and final income distribution for the country of interest. These vectors are such that individual income are ordered in increasing order. For the sake of simplicity, we suppose that the size of $\boldsymbol{X}$ and $\boldsymbol{Y}$ is identical and equal to n.[8] When the chosen poverty measure Θ complies with the scale invariance axiom — the value of the poverty measure is then insensitive to any change in the monetary unit —, the growth and inequality effects that ensues from Datt and Ravallion's (1992) approach are:

$$C_{\boldsymbol{X},\boldsymbol{Y}} = \Theta\left(z, \frac{\mu_{\boldsymbol{Y}}}{\mu_{\boldsymbol{X}}}\boldsymbol{X}\right) - \Theta\left(z, \boldsymbol{X}\right), \tag{B.1}$$

$$I_{\boldsymbol{X},\boldsymbol{Y}} = \Theta\left(z, \frac{\mu_{\boldsymbol{X}}}{\mu_{\boldsymbol{Y}}}\boldsymbol{Y}\right) - \Theta\left(z, \boldsymbol{X}\right). \tag{B.2}$$

where $\frac{\mu_{\boldsymbol{Y}}}{\mu_{\boldsymbol{X}}}\boldsymbol{X}$ is the income distribution characterized by the final value of income per capita but the Lorenz curve of the initial distribution. Conversely, $\frac{\mu_{\boldsymbol{X}}}{\mu_{\boldsymbol{Y}}}\boldsymbol{Y}$ is the income distribution that would exhibit the final degree of inequality but the initial mean income.

Now, consider the vector $\boldsymbol{\kappa}$ of the differences between $\boldsymbol{Y}$ and $\boldsymbol{X}$. This vector is then

[8] Generally, the income series are of different sizes. It is then necessary to transform them so as to get equally-sized vectors. The most common procedure consist in estimating the whole income distribution for each year using a kernel estimator. With the help of the predicted smoothed distributions, generating income vectors of the desired length becomes straightforward.

divided into s equal vectors in order to obtain s marginal variations in mean income and inequality degree. For each j-th addition of $s^{-1}\boldsymbol{\kappa}$ to the initial income vector, the equations (B.1) and (B.2) are applied so as to get the micro growth and inequality effects that correspond to a move from distribution $\boldsymbol{X}+\frac{j-1}{s}\boldsymbol{\kappa}$ to $\boldsymbol{X}+\frac{j}{s}\boldsymbol{\kappa}$, with $j\in\{1,2\ldots s\}$. Finally, the total growth and inequality effects are obtained by summing up of these micro-effects over the set $\{1,2\ldots s\}$. The empirical implementation of the general decomposition procedure defined in equation (3.6) is thus:

$$\hat{C}_{\boldsymbol{X},\boldsymbol{Y}}=\sum_{j=1}^{s}\Theta\left(z,\frac{\mu_j}{\mu_{j-1}}\left(\boldsymbol{X}+\frac{j-1}{s}\boldsymbol{\kappa}\right)\right)-\Theta\left(z,\boldsymbol{X}+\frac{j-1}{s}\boldsymbol{\kappa}\right), \tag{B.3}$$

$$\hat{I}_{\boldsymbol{X},\boldsymbol{Y}}=\sum_{j=1}^{s}\Theta\left(z,\frac{\mu_{j-1}}{\mu_j}\left(\boldsymbol{X}+\frac{j}{s}\boldsymbol{\kappa}\right)\right)-\Theta\left(z,\boldsymbol{X}+\frac{j-1}{s}\boldsymbol{\kappa}\right), \tag{B.4}$$

where $\mu_j=\mathrm{E}\left(X+\frac{j}{s}\boldsymbol{\kappa}\right)$.

References

AMIEL, Y., AND F. COWELL (1992): "Measurement of Income Inequality: Experimental Test by Questionnaire," *Journal of Public Economics*, 47, 3–26.

BAYE, F. M. (2006): "Growth, Redistribution and Poverty Changes in Cameroon: A Shapley Decomposition Analysis," *Journal of African Economies*, 15(4), 543–570.

BHANUMURTHY, N. R., AND A. MITRA (2004): "Economic Growth, Poverty, and Inequality in Indian States in the Pre-Reform and Reform Periods," *Asian Development Review*, 21(2), 79–99.

BRESSON, F., AND K. LABAR (2007): "'Leftist', 'rightist' and Intermediate Decompositions of Poverty Variations with an Application to China from 1990 to 2003.," Working Paper 76, ECINEQ.

DATT, G., AND M. RAVALLION (1992): "Growth and Redistribution Components of Changes in Poverty Measures: A Decomposition with Applications to Brazil and India in the 1980s," *Journal of Development Economics*, 38, 275–295.

DHONGDE, S. (2007): "Measuring the Impact of Growth and Income Distribution on Poverty in India," *Journal of Income Distribution*, 16(2), 25–48.

JAIN, L., AND S. TENDULKAR (1990): "The Role of Growth and Distribution in the Observed Change in Head-Count Ratio-Measure of Poverty: A Decomposition Exercise for India," *Indian Economic Review*, 25(2), 165–205.

KAKWANI, N. (2000): "On Measuring Growth and Inequality Components of Poverty with Application to Thailand," *Journal of Quantitative Economics*, 16(1), 67–79.

KAKWANI, N., AND K. SUBBARAO (1990): "Rural Poverty and its Alleviation in India," *Economic and Political Weekly*, 31/06, A2–A16.

MULLER, A. (2006): "Clarifying Poverty Decompostion," Scandinavian Working Papers in Economics 217, Göteborg University.

SASTRE, M., AND A. TRANNOY (2002): "Shapley Inequality Decomposition by Factor Components," in *Inequality: Theory, Measurement and Applications*, ed. by P. Moyes, C. Seidl, and A. Shorrocks, vol. Supplément 9 of *Journal of Economics/Zeitschrift für Nationalökonomie*, pp. 51–89. Springer.

SHAPLEY, L. (1953): "A Value for n-person Games," in *Contributions to the Theory of Games II*, ed. by W. Kuhn, and A. Tucker, Annals of Mathematics Studies. Princeton University Press.

SHORROCKS, A. P. (1999): "Decomposition Procedure for Distributional Analysis: A Unified Framework Based on the Shapley Value," Working paper, University of Essex.

SON, H. H. (2003): "A New Poverty Decomposition," *Journal of Economic Inequality*, 1(2), 181–187.

TSUI, K.-Y. (1996): "Growth-Equity Decomposition of a Change in Poverty: An Axiomatic Approach," *Economic Letters*, 50, 417–423.

WAN, G. H., AND Y. ZHANG (2006): "The Impacts of Growth and Inequality on Rural Poverty in China," *Journal of Comparative Economics*, 34, 694–712.

ZHENG, B. (1997): "Aggregate Poverty Measures," *Journal of Economic Surveys*, 11(2), 123–162.