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This paper aims to study the stability properties of a two-period overlapping generations model (OLG) with a progressive labor-income taxation rule. In this case, wage income tax rates are increasing with agent's income. Each representative agent lives two periods: youth and adulthood. In the first period, agents choose labor supply and allocate their after-tax income between consumptions and savings (capital accumulations). In the second period, agents are retired and consume entirely their savings returns. It is shown that progressive labor-income taxation policy acts as a destabilizing factor in the sense that a higher progressivity makes the emergence of indeterminacy and endogenous fluctuations more likely. These fluctuations appear if the elasticity of capital-labor substitution is sufficiently low. Moreover, we show that saving rate widens the range of parameters giving rise to endogenous fluctuations. The analytical findings are completed by a numerical example.

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Abstract

This paper aims to study the stability properties of a two-period overlapping generations model (OLG) with a progressive labor-income taxation rule. In this case, wage income tax rates are increasing with agent's income. Each representative agent lives two periods: youth and adulthood. In the first period, agents choose labor supply and allocate their after-tax income between consumptions and savings (capital accumulations). In the second period, agents are retired and consume entirely their savings returns. It is shown that progressive labor-income taxation policy acts as a destabilizing factor in the sense that a higher progressivity makes the emergence of indeterminacy and endogenous fluctuations more likely. These fluctuations appear if the elasticity of capital-labor substitution is sufficiently low. Moreover, we show that saving rate widens the range of parameters giving rise to endogenous fluctuations. The analytical findings are completed by a numerical example.

Key words: Progressive income taxes; Indeterminacy; Overlapping generations; Endogenous labor supply.

JEL classification: C62, E20, E32, H20.

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1 Introduction

A growing recent literature is concerned with studying the relationship between competitive equilibria stability and government fiscal policy. Particularly, it addresses the role of taxation policy on the appearance of endogenous fluctuations and local indeterminacy.

In an overlapping generations framework (thereafter OLG)[1], Dromel and Pintus (2006) (thereafter D-P (2006)) focus on a non-linear capital-income taxation rule in an OLG model à la Reichlin (1986). They show that endogenous fluctuations are ruled out and saddle-point stability ensured whenever tax progressivity is sufficiently high. However, Seegmuller (2003) proposes a constant tax policy, while Chen and Zhang (2009a, 2009b) and Gokan (2009a) consider an endogenous linear tax jointly with a two-period consumption OLG model. In particular, Chen and Zhang (2009a) concentrate on taxation on labor income while Chen and Zhang (2009b) study the effect of capital-income tax rule on the appearance of endogenous cycles. They deduce that labor-income tax policy (resp. capital-income tax policy) operates as a destabilizing factor (resp. stabilizing factor). These contradictory results can be intuitively explained as labor income taxation makes consumption-saving ratio smaller for young agents which makes cycles more likely to emerge.

In this paper, we complete previous analysis by asking whether the instability effect of labor-tax policy extends to a tax policy with progressive feature. This policy is consistent with the US tax code in which labor-income taxation is more progressive than capital-income taxation (See Hall and Rabushka, 1995). According to above literature, it is shown that the stability effect of taxation rule relies on the tax base (capital or labor income). As mentioned by D-P (2006), progressive fiscal policy has a stabilization power if taxed income finances all or at least most of consumptions generated by agents. Hence, first-period consumption[2]might be a crucial element for the stability effect of fiscal rules in an OLG economy.

This paper extends the work of Chen and Zhang (2009a) by considering a progressive labor income tax in an OLG with two-period consumptions and aims to study the influence of tax progressivity on economic stability. At the same time, this work allows to study the robustness of existing results obtained by D-P (2006). Progressive tax is measured whenever the marginal tax rate is larger than the average tax rate. Government purchases which are financed only through agent's income tax do not contribute to either production or agent's utility.

Contrary to D-P (2006), this work shows that high progressive labor-income taxes performs as a *destabilizing factor* in the sense that it promotes endogenous cycles. These cycles arise due to the presence of interaction of two conflicting

[1] The role of tax-rate progressivity is also treated in a heterogenous agents framework. For instance, in a segmented agents model à la Woodford (1986), see among others, Dromel and Pintus (2008) and Lloyd-Braga, Modesto and Seegmuller (2008), however, heterogeneity à la Becker (1980) is considered by Sarte (1997), Sorger (2002) and Bosi and Seegmuller (2010).

[2] See Cazzavillan and Pintus (2004).

effects on savings that operate through wages and real interest rate. The intuition goes as follows: assume that current capital increases from its stationary level. This implies an increase in wages and so more capital accumulations. At the same time, agents expect that an augmentation in future capital reduces future interest rate. Such an expectation induces them to supply less labor which in turn has a negative effect on capital accumulation. Cyclical paths emerge whenever the interest rate effect dominates the wage effect. This requires a sufficiently low elasticity of input substitution together with a high propensity to save out of after-tax wage income and a high tax progressivity.

Therefore, while taxation policy acts as a destabilizing factor in our model, it has a stabilizing power in D-P (2006). Such a difference is due to that, in this paper, taxes are imposed on labor income and agents consume in both periods. On the contrary, D-P (2006) assume that taxes are imposed on capital income and agents consume only when they are old. These assumptions are crucial to give rise to different results from a stability point of view. In this study, indeterminacy requires high saving rate which means that a low fraction of consumptions is financed by labor income. However, in D-P (2006), consumptions are totally financed by capital income which confirms their principle conclusion: if agent's consumptions are mostly financed by wage income (resp. capital income), then the steady state exhibits a stable saddle-path if progressive taxes are applied to labor income (resp. capital income). Likewise, progressive labor income tax is not helpful to stabilize both labor and consumptions since second-period consumption depends strictly on the interest rate.

The dynamic effects of taxation policies have also been studied in a one-sector infinite horizon representative agent model. The pioneering paper of Schmitt-Grohé and Uribe (1997) considers a balanced-budget rule with a constant returns-to-scale technology where a *constant* government spending is financed by fixed income taxes. This countercyclical tax policy can generate indeterminate steady state and a continuum of sunspots equilibria. However, Guo and Harrison (2004) extend Schmitt-Grohé-Uribe model by allowing endogenous government expenditures financed by proportional tax rates on capital and labor incomes. They show that indeterminacy is no longer appear and the economy exhibits saddle-path stability. Moreover, Guo and Lansing (1998) find that introduction a progressive income tax in a model with increasing returns as in Benhabib and Farmer (1994) can prevent agents' self-fulfilling expectations and provides a unique equilibrium. More recently, Dromel and Pintus (2007) consider a linear income-tax policy into the model of Benhabib and Farmer (1994) where it is assumed that this tax is constant and applied to income only when the latter is higher than a threshold value. They demonstrate that taxes can prevent the occurrence of endogenous fluctuations. The stabilizing role of the progressive tax can be interpreted as follows: when agents are optimistic and expect a higher wage tomorrow and simultaneously they realize to face an increasing tax rate. This reduces their expected after-tax returns and prevent the occurrence of self-fulfilling expectation.

The outline of this paper is the following: Section 2 presents the model. Section 3 characterizes the intertemporal equilibrium. Section 4 demonstrates the existence of a steady state. The local dynamics is introduced in section 5 followed by a numerical example in section 6 and finally section 7 concludes.

2 The model

We consider a competitive, non monetary, two-period overlapping generations model with identical agents. In each period t, N_t individuals are born and live only two periods "young and old". There exists a unique good which can be either consumed or saved. In the first period, agents allocate consumption and saving according to their after-tax income and in the next period they do not supply labor and consume the entire first-period saving returns. In addition, agents are imposed to pay a labor income tax which is supposed to be progressive.

Given the real wage w_t and the gross real interest rate R_{t+1}, an agent born at time $t \geq 0$ choose labor l_t, saving s_t and both period consumptions (c_t, d_{t+1}) to maximize the following additive separable preferences:

$$\max_{c_t,\ d_{t+1},\ l_t} \left[u\left(c_t/B\right) + \beta u\left(d_{t+1}\right) - v\left(l_t\right)\right] \tag{1}$$

subject to

$$\begin{aligned} c_t + s_t &= \varphi\left(w_t l_t\right) && (2)\\ d_{t+1} &= R_{t+1} s_t && (3)\\ c_t &\geq 0,\ d_{t+1} \geq 0 \text{ for all } t \geq 0 \end{aligned}$$

with $\varphi\left(w_t l_t\right) = w_t l_t - \tau\left(w_t l_t\right)$ is the after-tax income and $\tau\left(w_t l_t\right)$ is the amount of tax payment, $\beta \in (0,1)$ is the discount factor and B is a scaling parameter.

Assumption 1 *$u(x)$ is continuous, increasing $u'(x) > 0$ and concave $u''(x) < 0$ for $x = c_t/B, d_{t+1}$, while $v(l)$ is continuous for $0 < l < \zeta$ and increasing $v'(l) > 0$ and convex $v''(l) > 0$. Additionally, $\lim_{x\to+\infty} u'(x) = 0$, $\lim_{x\to 0^+} u'(x) = +\infty$, $\lim_{l\to 0^+} v'(l) = 0$ and $\lim_{l\to\zeta} v'(l) = +\infty$.*

For future reference, we present the following necessary elasticities: the elasticity of marginal utility of current and future consumption $\varepsilon_{11} = u''(x)\,x/u'(x) < 0$ for $x = c_t/B, d_{t+1}$ and the elasticity of marginal disutility of labor $\varepsilon_v = v''(l_t)\,l_t/v'(l_t) > 0$.

Assumption 2 *The function $\varphi\left(w_t l_t\right) \in C^2$ is positive with $\varphi(0) = 0$. Further, it satisfies $0 < \varphi'\left(w_t l_t\right) \leq 1$, $\varphi''\left(w_t l_t\right) < 0$ for all $w_t l_t > 0$. Moreover, the income tax is a progressive tax, that is, $\varphi\left(w_t l_t\right)/w_t l_t$ is non-increasing for $w_t l_t > 0$ or equivalently $\varphi'\left(w_t l_t\right) w_t l_t/\varphi\left(w_t l_t\right) \leq 1$.*[3]

[3] For details, see among others Dromel and Pintus (2006) and Bosi and Seegmuller (2010).

The Lagrangian function for household problem is:

$$Lag = u\left(c_t/B\right) + \beta u\left(d_{t+1}\right) - v\left(l_t\right) + \lambda_t\left(\varphi\left(w_t l_t\right) - c_t - s_t\right) + \mu_t\left(R_{t+1}s_t - d_{t+1}\right) \tag{4}$$

The first-order conditions with respect to c_t, d_{t+1}, l_t and s_t imply:

$$Bv'\left(l_t\right) = u'\left(c_t/B\right) w_t \varphi'\left(w_t l_t\right) \tag{5}$$

$$v'\left(l_t\right) = \beta u'\left(d_{t+1}\right) R_{t+1} w_t \varphi'\left(w_t l_t\right) \tag{6}$$

Income tax progressivity can be obtained whenever the marginal income tax rate is higher than the average tax rate. More explicitly, the marginal tax rate is $\tau'\left(w_t l_t\right) = 1 - \varphi'\left(w_t l_t\right)$ and the average tax rate is $\tau\left(w_t l_t\right)/w_t l_t = 1 - \varphi\left(w_t l_t\right)/w_t l_t$. Thus, progressive taxation requires marginal rate greater than average rate, i.e., $\varphi'\left(w_t l_t\right) \leq \varphi\left(w_t l_t\right)/w_t l_t$. This fiscal policy generalizes Chen and Zhang (2009a) where they suppose an endogenous linear taxation policy. The corresponding elasticities of after-tax income are:

$$\left(\eta_1, \eta_2\right) \equiv \left(\frac{\varphi'\left(wl\right)wl}{\varphi\left(wl\right)}, \frac{\varphi''\left(wl\right)wl}{\varphi'\left(wl\right)}\right) \tag{7}$$

where $\eta_1 \in (0,1]$ and $\eta_2 \leq 0$ are respectively the first-order and the second-order elasticity of after-tax income with respect to labor income.[4]

2.1 Firms

On the production side, firms which are identical utilize capital K_t and labor L_t to produce final goods and to maximize the profit:

$$\pi_t = AF\left(K_t, L_t\right) - R_t K_t - w_t L_t$$

with $F\left(K_t, L_t\right)$ is the production function which has the following features.

Assumption 3 *$F\left(K,L\right)$ is a continuous function defined on $[0,+\infty)$, homogeneous of degree one, strictly increasing in both arguments $\left(F_K\left(K,L\right) > 0, F_L\left(K,L\right) > 0\right)$ and strictly concave $\left(F_{KK}\left(K,L\right) < 0, F_{LL}\left(K,L\right) < 0\right)$.*[5] *Additionally, $F\left(0,0\right) = 0$ and the boundary conditions $\lim_{k\to 0^+} f'\left(k\right) = +\infty$, $\lim_{k\to+\infty} f'\left(k\right) = 0^+$ are satisfied, where $f\left(k\right) \equiv F\left(k,1\right)$ is the production per labor and $k \equiv K/L$ is the capital-labor ratio.*

If we set $\rho\left(k\right) \equiv f'\left(k\right)$ and $\omega\left(k\right) = f\left(k\right) - kf'\left(k\right)$ we then obtain:

$$R\left(k\right) = A\rho\left(k\right) \text{ and } w\left(k\right) = A\omega\left(k\right) \tag{8}$$

As a result, the elasticity of interest rate $k\rho'\left(k\right)/\rho\left(k\right) = -\left(1-s\right)/\sigma < 0$ and the elasticity of real wage $\omega'\left(k\right)k/\omega\left(k\right) = s/\sigma > 0$, with $s \in (0,1)$ is the capital share in total income and $\sigma \in (0,+\infty)$ is the elasticity of capital-labor substitution.

[4] In this paper, tax progressivity is defined in marginal terms as implemented by Musgrave and Thin (1948).

[5] Notice that: $F_L\left(K,L\right) \equiv \partial F\left(K,L\right)/\partial L$ and $F_K\left(K,L\right) \equiv \partial F\left(K,L\right)/\partial K$. This notation holds across the paper.

2.2 Government

As in Schmitt-Grohé and Uribe (1997), Guo and Lansing (1998) and Bosi and Seegmuller (2010) labor income taxes are used to finance government expenditure G_t. Hence, the instantaneous government budget constraint is:

$$G_t = N_t \tau (w_t l_t) \tag{9}$$

Notice that public spending does not contribute to either production or household utility.

3 Intertemporal equilibrium

The number of households at each generation grows at a constant rate $n > -1$ such that $1 + n = N_{t+1}/N_t$ where N_t is the number of population born at time t. At equilibrium, all markets clears:

1. Capital market clears according to capital-accumulation equation: $N_t s_t = K_{t+1}$.
2. Labor market clears: $L_t = N_t l_t$.
3. Government spending G_t is determined by the balanced budget rule (9).
4. By Walras' law, output market also clears: $N_t (c_t + s_t) + N_{t-1} d_t + G_t = AF(K_t, L_t)$.

From market clearing conditions, one can easily demonstrate that:

$$s_t = k_{t+1} (1 + n) l_{t+1} \tag{10}$$

Substituting (10) and (8) together with conditions (2)-(3) in the first-order conditions (5) and (6) yields the following two-dimensional dynamic system of k and l.

$$Bv'(l_t) = u' \left[\frac{\varphi (A\omega (k_t) l_t) - k_{t+1} (1+n) l_{t+1}}{B} \right] A\omega (k_t) \varphi' (A\omega (k_t) l_t) \tag{11}$$

$$v'(l_t) = \beta u' [A\rho (k_{t+1}) k_{t+1} (1+n) l_{t+1}] A\rho (k_{t+1}) A\omega (k_t) \varphi' (A\omega (k_t) l_t) \tag{12}$$

4 The steady state

The steady state of the dynamic system (11)-(12) is a solution (k, l) of the system:

$$Bv'(l) = u' \left[\frac{\varphi (A\omega (k) l) - k (1+n) l}{B} \right] A\omega (k) \varphi' (A\omega (k) l) \tag{13}$$

$$v'(l) = \beta u' [A\rho (k) k (1+n) l] A\rho (k) A\omega (k) \varphi' (A\omega (k) l) \tag{14}$$

Obviously, there might exists one or multiple steady state for the system (13)-(14). However, to simplify the analysis, we follow Cazzavillan, Lloyd-Braga and Pintus (1998) by showing the existence of a normalized steady state such that $(k, l) = (1, 1)$ by selecting suitably the scaling parameters A, $B > 0$.[6]

Assumption 4 $1 + \varepsilon_{11} > 0$.

This assumption has been made by, among others, Cazzavillan and Pintus (2004), Bosi and Seegmuller (2008) in order to ensure that consumptions in both periods and leisure are substitutable goods and saving function is increasing in the gross rate of return R.

Proposition 1 *Let Assumptions* $1-4$ *be satisfied, there exists a steady state of dynamic system (11)-(12) such that* $k = 1$ *and* $l = 1$ *for one of these cases:*

1. *If the following sufficient boundary conditions are satisfied:*

$$\lim_{A \to \underline{A}} \beta u' \left[A\rho(1)(1+n) \right] A\rho(1) A\omega(1) \varphi'(A\omega(1)) < v'(l) \quad (15)$$

$$\lim_{A \to +\infty} \beta u' \left[A\rho(1)(1+n) \right] A\rho(1) A\omega(1) \varphi'(A\omega(1)) > v'(l) \quad (16)$$

   then, $\exists\ A^* > \underline{A} \equiv \varphi^{-1}(1+n)/\omega(1)$ *such that (14) is verified at* $(k, l) = (1, 1)$. *However, if at* $(k, l) = (1, 1)$, *we have* $1 + \varepsilon_{11} + \eta_2 + 1 > 0$ *for all* A, *then* A^* *is unique (because of the continuity of functions involved in (14), i.e., the functions* $\{v, u, F, \varphi\} \in C^2$*).*

2. *If the following sufficient boundary conditions are satisfied:*

$$\lim_{A \to \underline{A}} \beta u' \left[A\rho(1)(1+n) \right] A\rho(1) A\omega(1) \varphi'(A\omega(1)) > v'(l) \quad (17)$$

$$\lim_{A \to +\infty} \beta u' \left[A\rho(1)(1+n) \right] A\rho(1) A\omega(1) \varphi'(A\omega(1)) < v'(l) \quad (18)$$

   then, $\exists\ A^* > \underline{A} \equiv \varphi^{-1}(1+n)/\omega(1)$ *such that (14) is verified at* $(k, l) = (1, 1)$. *However, if at* $(k, l) = (1, 1)$, *we have* $1 + \varepsilon_{11} + \eta_2 + 1 < 0$ *for all* A, *then* A^* *is unique (because of the continuity of functions involved in (14), i.e., the functions* $\{v, u, F, \varphi\} \in C^2$*).*

   Moreover, given the value of A^*, *there exists* $B^* > 0$ *which is a unique solution of (13) at* $(k, l) = (1, 1)$.

Proof. See Appendix (A). ■

Corollary 1 *Suppose that the after-tax income function* $\varphi(wl)$ *is locally isoelastic, then the boundary conditions (15)-(16) are sufficient for the existence of a unique* $A^* > \underline{A}$.

Simply, if $\varphi(wl)$ has an isoelastic formulation around the steady state, then one can show that $\eta_2 = \eta_1 - 1$. As a result and according to Assumption 4 we have $1 + \varepsilon_{11} + \eta_2 + 1 > 0$.

[6] For simplicity, we concentrate on local dynamics around the normalized steady state without illustrating the possible existence of other steady state.

5 Local dynamics

In this section, we study the role of income taxation on the occurrence of endogenous fluctuations due to self-fulfilling prophecies. It is supposed that the after-tax income function $\varphi(w_t l_t)$ is locally isoelastic which provides that $\eta_1 = 1+\eta_2$. Linearizing the dynamic equation (11)-(12) around the normalized steady state $k = l = 1$ yields the following Jacobian matrix J:

$$J = \begin{bmatrix} \varepsilon_{11}\gamma/(1-\gamma) & \varepsilon_{11}\gamma/(1-\gamma) \\ \varepsilon_{11} - (1-s)(1+\varepsilon_{11})/\sigma & \varepsilon_{11} \end{bmatrix}^{-1} \begin{bmatrix} Z_2 & Z_1 \\ -s(1+\eta_2)/\sigma & \varepsilon_v - \eta_2 \end{bmatrix} \tag{19}$$

with $Z_1 \equiv \varepsilon_{11}(1+\eta_2)/(1-\gamma)+\eta_2-\varepsilon_v$ and $Z_2 \equiv s(1+\eta_2)(\varepsilon_{11}/(1-\gamma)+1)/\sigma$. The characteristic polynomial of (19) is $P(\lambda) = \lambda^2 - T\lambda + D$, where the trace $T = \lambda_1 + \lambda_2$ and the determinant $D = \lambda_1\lambda_2$ are respectively given by:

$$T = \frac{X_1 + X_2 X_3}{\varepsilon_{11}\gamma(1-s)(1+\varepsilon_{11})} > 0 \tag{20}$$

$$D = \frac{s(1+\eta_2)(1+\varepsilon_v)}{\gamma(1-s)(1+\varepsilon_{11})} > 0 \tag{21}$$

with $X_1 \equiv \gamma\sigma\varepsilon_{11}(\varepsilon_v - \eta_2) + s\varepsilon_{11}(1+\eta_2)(1+\varepsilon_{11})$, $X_2 \equiv (1-s)(1+\varepsilon_{11}) - \sigma\varepsilon_{11}$ and $X_3 \equiv (\eta_2 - \varepsilon_v)(1-\gamma) + \varepsilon_{11}(\eta_2+1)$.

In this model, l_t is an independently non-predetermined variable, this means that the steady state is locally indeterminate if and only if both eigenvalues are located within a unit circle, i.e., $\lambda_1,\lambda_2 \in (-1,1)$. Using the fact that the trace T and the determinant D are respectively the sum and the product of the eigenvalues, then local indeterminacy requires that $D < 1$ and $T < 1 + D$.

Therefore, we study analytically the conditions where the above inequalities are satisfied using the following parameters σ, ε_{11}, γ, η_2 and ε_v. As previously defined that σ is the elasticity of capital-labor substitution, ε_{11} is the elasticity of marginal utility of consumption, γ is the propensity to save (saving over after-tax labor income), η_2 is the elasticity of marginal after-tax income, ε_v is the elasticity of labor supply.

We present the critical values for η_2, ε_{11}, γ, σ and ε_v in Appendix (B). Our main results are summarized in the following proposition:

Proposition 2 *Given that* $\gamma > \max\{\gamma^*, \gamma^D\}$ *and* $\sigma < \sigma^*$ *together with Assumptions* $1-4$, *local indeterminacy emerges at the following conditions:*

1. $-1 < \varepsilon_{11} < \varepsilon_{11}^D$ *for all* ε_v *and* $\eta_2 < \eta_2^D$.
2. $\varepsilon_{11}^D < \varepsilon_{11} < 0$ *with either* $\varepsilon_v < \varepsilon_v^D$, *all* η_2 *or* $\varepsilon_v > \varepsilon_v^D$ *and* $\eta_2 < \eta_2^D$.

Proof. See Appendix C. ■

Mainly, Proposition (2) states that endogenous fluctuations require a sufficiently low elasticity of capital-labor substitution and high saving rates. Moreover, the presence of high progressive taxations (η_2 is sufficiently low) enforces the appearance of indeterminacy.

6 Discussions

In order to complete the characterization of economic stability, we provide the economic interpretations for local indeterminacy and show how the presence of progressive tax influences the appearance of endogenous cycles.

Before going through the economic intuition, we need to do the following computations: using (8) and the elasticity of interest rate $k\rho'(k)/\rho(k) = -(1-s)/\sigma$, we get:

$$\frac{\partial R_{t+1}}{R_{t+1}} = -\frac{1-s}{\sigma}\frac{\partial K_{t+1}}{K_{t+1}}, \frac{\partial R_{t+1}}{R_{t+1}} = \frac{1-s}{\sigma}\frac{\partial L_{t+1}}{L_{t+1}} \tag{22}$$

$$\frac{\partial l_t}{l_t} = \frac{1+\eta_2}{\varepsilon_v - \eta_2}\frac{\partial w_t}{w_t}, \frac{\partial l_t}{l_t} = \frac{1+\varepsilon_{11}}{\varepsilon_v - \eta_2}\frac{\partial R_{t+1}}{R_{t+1}} \tag{23}$$

The model of Cazzavillan and Pintus (2004) is recovered by setting zero labor income taxes, i.e., $\eta_2 = 0$. The objective of this section is to show how a high tax progressivity promotes indeterminacy. Endogenous fluctuations arise due to the presence of interaction of two conflicting effects on savings that operate through wages and real interest rate.

Let us start by the Benchmark model where $\eta_2 = 0$. Assume at time t an instantaneous increase in current capital stock K_t from its steady state. This generates two opposite effects through wages and interest rate: an increase in K_t implies a rise in the wage income, and according to (23), agents supply more labor. Given the budget constraint of young agents (2), a rise in wage income implies a rise in K_{t+1}.

The second effect is the anticipation effect that plays in the opposite direction in the sense that a higher K_{t+1} is followed by a decline in the interest rate R_{t+1}. Given that $(\partial l_t/l_t)/(\partial R_{t+1}/R_{t+1}) = (1+\varepsilon_{11})/\varepsilon_v$, then a decrease in R_{t+1} is followed by a decline in the labor supply and according to the budget constraint, a low accumulation of capital K_{t+1} occurs. A cyclical path emerges whenever anticipation effect dominates current effect. This setting requires a sufficiently low elasticity of capital-labor substitution σ as shown in (22) and a low elasticity of marginal disutility of labor ε_v as shown in (23). Indeed, equality (23) shows that the interest rate has a strong influence on current labor supply for sufficiently high elasticity of intertemporal substitution in consumption.

In the presence of a progressive tax[7], equation (23) shows that a higher progressivity ($\eta_2 \longrightarrow -1$) implies a lower elastic labor supply to both expected interest rate and real wage. In particular, when η_2 is close enough to -1, wages do not have any influence on labor supply (see (23)). As a result, this makes the anticipation effect more dominant than the current effect and so deterministic cycles are more likely to appear. Therefore, progressive taxes act as a destabilizing factor. This result is reasonable because introducing progressive taxes on labor income can stabilize wage income only. However, this is not enough to stabilize future consumptions since the latter depends on the capital real returns.

[7] Notice that the presence of a progressive tax implies that η_2 declines from 0 to -1.

Above results are almost similar to those obtained by Chen and Zhang (2009a) with linear tax policy. Conversely, D-P (2006) consider a model where agents can consume only in the second period and save in the first period. They demonstrate that progressive taxation of capital income performs as a stabilizer factor. Intuitively, our results are in line with Dromel and Pintus in the sense that tax progressivity does not perform as a stabilizer if consumptions are partially financed by a non-taxed capital income.

Finally, in order to study the effect of the saving rate on steady state stability, consider the bifurcation value of $\sigma = \sigma^*$ given by (30) in the Appendix and differentiate it with respect to γ, we obtain

$$\frac{\partial \sigma^*}{\partial \gamma} = \frac{(1-s)(1+\varepsilon_{11})}{\varepsilon_{11}} \frac{\varepsilon_{11} + \eta_2 - \varepsilon_v}{\varepsilon_v - \eta_2 - \varepsilon_{11}(\eta_2 + 1)} > 0 \tag{24}$$

Therefore, a higher saving rate makes the emergence of local indeterminacy more likely. This result is analogous to Cazzavillan and Pintus (2004) after setting $\eta_2 = 0$. Therefore, in their model, the necessary condition for local indeterminacy becomes $\sigma < \sigma^*_{C,P}$ with

$$\sigma^*_{C,P} \equiv -\frac{s + (1+\varepsilon_{11})((1-s)\gamma - 1)}{\varepsilon_{11}}$$

The positivity of $\sigma^*_{C,P}$ requires a sufficiently high saving rate, that is, $\gamma > \gamma_{C,P}$ with $\gamma_{C,P} \equiv (1+\varepsilon_{11} - s) / (1+\varepsilon_{11})(1-s)$. Additionally, one can easily find that $\sigma^*_{C,P} < s$ and $\partial \sigma^*_{C,P} / \partial \gamma > 0$. Thus, endogenous fluctuations necessitate a low elasticity of input substitution (complementary inputs) and a high propensity to save.

7 A numerical illustration

In this example, we present numerically the analytical results obtained in Proposition (2). To do that, consider the following explicit formulas for the production function, the preference and the after-tax income function respectively:

$$y = Af(k) = A\left[sk^{-\zeta} + (1-s)\right]^{-1/\zeta} \tag{25}$$

$$u(x) = \frac{x^{1-\varepsilon}}{1-\varepsilon} \text{ and } v(l) = \frac{l^{1+\varkappa}}{1+\varkappa} \text{ with } x = \frac{c}{B}, d \tag{26}$$

$$\varphi(w_t l_t) = (w_t l_t)^{1+\eta_2} \tag{27}$$

with $A > 0$ is the a scaling parameter, $s \in (0,1)$, $\eta_2 \in (-1,0)$, $\varepsilon \in (0,1)$, $\varkappa > 0$ and $\zeta > -1$ with $\zeta \neq 0$, $n = 0.5175$ and $\beta = 0.3$. Further, one can easy show that the elasticity of capital-labor substitution is $1/(1+\zeta)$, the elasticity of marginal utility of consumption is $\varepsilon_{11} = -\varepsilon < 0$ which is the inverse of the elasticity of intertemporal substitution in consumption $-1/\varepsilon$, the elasticity of marginal labor supply is simply $\varepsilon_v = \varkappa$ and the first-order elasticity of after-tax income is $1 + \eta_2 \in (0,1)$.

Given the normalized steady state $(k,l)=(1,1)$ then, capital share in total income does not depend on the elasticity of capital-labor substitution, thus $s\in(0,1)$. Let us set $s=1/3$, then according to Proposition (2) one can show that the critical value $\varepsilon_{11}^{D}=-1/2$. Consistent with case (1) of Proposition (2), choose $\varepsilon=0.55$ and $\varkappa=1$ we obtain $\eta_2^D=-0.55$. Therefore, consider $\eta_2=-0.8$, we can easily compute the lower bound of the propensity to save values $\gamma^D=0.444\,44$ and $\gamma^*=0.732\,15$. Since $\gamma>\max\left\{\gamma^D,\gamma^*\right\}$, then choose $\gamma=0.74$, we get $\sigma^*=5.\,267\,3\times10^{-3}$. This model has a one predetermined variable, so that, indeterminacy appears if and only if both eigenvalues of Jacobian matrix (19) belong to a unit cycle, i.e., $\lambda_1,\lambda_2\in(0,1)$. Therefore, setting $\sigma=0.005$, we show that endogenous cycles emerge for above critical values with $\lambda_1=0.994\,19\in(0,1)$ and $\lambda_2=0.604\,11\in(0,1)$. Finally, given the above values, we compute the scaling parameter value $A=12.\,134>\underline{A}=12.\,071$ which is necessary for normalizing the steady state.

Concerning the case (2) of Proposition (2), choose $\varepsilon=-0.4$ we obtain that $\varepsilon_v^D=0.2$. Choose again $\varepsilon_v=0.1$ and $\eta_2=-0.5$, we can compute that $\gamma^D=0.458\,33$ and $\gamma^*=0.716\,67$. Thus, set $\gamma=0.72$ we deduce the upper bound of the elasticity of capital-labor substitution $\sigma^*=4.\,166\,7\times10^{-3}$, as a result, for $\sigma=0.004$, endogenous cycles appear with eigenvalues: $\lambda_1=0.998\,72$ and $\lambda_2=0.637\,39$ and the scaling parameter $A=6.\,214\,5>3.\,454\,2=\underline{A}$.

The second conditions of case (2) are $\varepsilon_v>\varepsilon_v^D$ together with $\eta_2<\eta_2^D$. Let us set $\varepsilon_v=0.8$, then we get $\eta_2^D=-0.333\,33$. Therefore, choose $\eta_2=-0.7$, then the saving rate values are $\gamma^D=0.45$ and $\gamma^*=0.789\,47$. If we consider $\gamma=0.8$, then $\sigma^*=1.234\,6\times10^{-2}$, so that, indeterminacy is obtained for $\sigma=0.01$ with the eigenvalues: $\lambda_1=0.971\,81$ and $\lambda_2=0.578\,82$ with $A=9.\,920\,4>6.\,023\,6=\underline{A}$.

According to above numerical example, it is shown that local indeterminacy is obtained for non-plausible values of saving rate γ.[8] Moreover, one can observe that endogenous cycles require a sufficiently low elasticity of capital-labor substitution close to the Leontief case. However, for realistic values of consumption-wage ratio, around 65%, indeterminacy is no longer appear and the equilibria is locally unique and converges to the steady state.

8 Conclusion

We have studied the effect of a progressive labor-income tax policy on steady state properties in an OLG model where agents can consume in both periods (young and old). It is supposed separable preferences with respect to current-period consumption, future-period consumption and labor supply. Agents have to pay taxes related to their wage income. Further, the production function exhibits a constant return-to-scale property. It is shown that progressive taxes act as a destabilizing factor where it makes endogenous fluctuation more likely. Similar to previous literature, indeterminacy occurs for sufficiently low elasticity

[8] The annual ratio of personal consumption expenditures over GDP have an average of 0.65 over the period (1959 - 2008) for US economy (see Economic Report of the President, 2008).

of factor substitution close to Leontief and non-realistic high values of young's saving rate.

9 Appendix

(A) Proof of Proposition 1. **Proof.** Consider (13) and (14) at the normalized steady state $(1,1)$, we get:

$$v'(1) = \frac{1}{B}u'\left[\frac{\varphi(A\omega(1))-(1+n)}{B}\right]A\omega(1)\varphi'(A\omega(1)) \quad (28)$$

$$v'(1) = \beta u'[A\rho(1)(1+n)]A\rho(1)A\omega(1)\varphi'(A\omega(1)) \quad (29)$$

Let us start with equality (29): LHS is a positive constant but the RHS is either increasing or decreasing in A. If we denote the RHS as $Q(A)$, then one can show that $\partial Q(A)/\partial A = 1+\varepsilon_{11}+\eta_2+1$ which depends on A. Whenever $\partial Q(A)/\partial A > 0$ (resp. < 0), then the RHS of (29) is strictly increasing (resp. decreasing) in A. Moreover, the positivity of first period consumption requires $A > \underline{A} \equiv \varphi^{-1}(1+n)/\omega(1)$.

Let us firstly focus on the case where $\partial Q(A)/\partial A > 0$. In this case, the existence of a value A^* that solves (29) requires the following boundary conditions: $\lim_{A\to\underline{A}} RHS < v'(l)$ and $\lim_{A\to+\infty} RHS > v'(l)$. Notice that if $\partial Q(A)/\partial A > 0$ for all A, then A^* is unique.

However, when $\partial Q(A)/\partial A < 0$, the existence of A^* which solves (29) necessitates initially $\lim_{A\to\underline{A}} RHS > v'(l)$ and further $\lim_{A\to+\infty} RHS < v'(l)$. If $\partial Q(A)/\partial A < 0$ for all A, then A^* is unique.

Given $A = A^*$, then from (28), the LHS is a positive constant and the RHS is decreasing in B according to Assumption 4. In addition, $\lim_{B\to 0^+} RHS = +\infty$ and $\lim_{B\to+\infty} RHS = 0^+$, therefore, there exists a unique $B^* > 0$ that solves (28).

Consequently, we have shown that there are unique $A^* > \underline{A}$ and $B^* > 0$ such that $(k,l) = (1,1)$ is a steady state of the system (11)-(12). ■

(B) Critical values. Here, we present the critical values of the model:

$$\sigma^* \equiv \frac{\varepsilon_{11}(1+\eta_2)[s(1+\varepsilon_v)-(1+\varepsilon_{11})]+(1-s)(1+\varepsilon_{11})[(\varepsilon_v-\eta_2)(1-\gamma)+\varepsilon_{11}\gamma]}{\varepsilon_{11}[\varepsilon_v-\eta_2-\varepsilon_{11}(\eta_2+1)]} \quad (30)$$

$$\gamma^* \equiv \frac{\varepsilon_{11}(1+\eta_2)(1+\varepsilon_{11}-s(1+\varepsilon_v))-(1-s)(1+\varepsilon_{11})(\varepsilon_v-\eta_2)}{(\varepsilon_{11}+\eta_2-\varepsilon_v)(1-s)(1+\varepsilon_{11})} \quad (31)$$

$$\gamma^D \equiv \frac{s(1+\eta_2)(1+\varepsilon_v)}{(1-s)(1+\varepsilon_{11})} \quad (32)$$

$$\eta_2^* \equiv -\frac{\varepsilon_{11}s(1+\varepsilon_v)+(1+\varepsilon_{11})((1-s)\varepsilon_v-\varepsilon_{11})}{\varepsilon_{11}s(1+\varepsilon_v)-(1+\varepsilon_{11})(1-s+\varepsilon_{11})} \quad (33)$$

$$\eta_2^D \equiv \frac{(1-s)(1+\varepsilon_{11})-s(1+\varepsilon_v)}{s(1+\varepsilon_v)} \quad (34)$$

$$\varepsilon_v^D \equiv \frac{(1-s)(1+\varepsilon_{11})-s}{s} \quad (35)$$

$$\varepsilon_{11}^D \equiv \frac{2s-1}{1-s} \quad (36)$$

$$\varepsilon_{11}^{**} \equiv s-1 \quad (37)$$

(C) Proof of Proposition 2. **Proof.** In this proposition, we show that the existence of endogenous fluctuations requires $D < 1$ and $T < 1 + D$. Let us begin with $D < 1$.

$D < 1$ if and only if $\gamma > \gamma^D$ with $\gamma^D > 0$. However, $\gamma^D < 1$ is met if and only if $\eta_2 < \eta_2^D$ with $\eta_2^D > -1$. Since η_2 is the second-order elasticity, then it should always be negative, so $\eta_2^D < 0$ if and only if $\varepsilon_v > \varepsilon_v^D$. One has to be sure that ε_v^D is positive, a direct inspection implies that for $\varepsilon_v^D > 0$ if and only if $\varepsilon_{11} > \varepsilon_{11}^D$.

As a result $D < 1$ for (1) $\varepsilon_{11} > \varepsilon_{11}^D$, $\gamma > \gamma^D$ and either $\varepsilon_v > \varepsilon_v^D$ and $\eta_2 < \eta_2^D$ or $\varepsilon_v < \varepsilon_v^D$, for all $\eta_2 < 0$. In addition, (2) $D < 1$ for $\varepsilon_{11} < \varepsilon_{11}^D$, $\eta_2 < \eta_2^D$ and $\gamma > \gamma^D$ for all $\varepsilon_v > 0$.

Furthermore, $T < 1 + D$ if and only if $\sigma < \sigma^*$. The latter inequality requires $\sigma^* > 0$ which is met for $\gamma > \gamma^*$ with $\gamma^* < 1$. The positivity of γ requires that $\gamma^* > 0$ and this inequality is satisfied if and only if $\eta_2 < \eta_2^*$. It is immediate to verify that $\eta_2^* > 0$ for $\varepsilon_{11} > \varepsilon_{11}^{**}$ and $\eta_2^* < 0$ for $\varepsilon_{11} < \varepsilon_{11}^{**}$. On the one hand, when $\eta_2^* > 0$, then $\gamma^* > 0$ for all $\eta_2 < 0$.

As a result, $T < 1 + D$ for $\sigma < \sigma^*$ and $\gamma > \gamma^*$ with $\varepsilon_{11} > \varepsilon_{11}^{**}$ for all $\eta_2 \in (-1, 0)$ and all values of $\varepsilon_v > 0$. On the other hand, whenever $\varepsilon_{11} < \varepsilon_{11}^{**}$ then $\eta_2^* \in (-1, 0)$ always. As a result, $T < 1 + D$ for $\sigma < \sigma^*$, $\varepsilon_{11} < \varepsilon_{11}^{**}$ and for all $\varepsilon_v > 0$ with *either* $\gamma > \gamma^*$ and $\eta_2 < \eta_2^*$ *or* $\gamma \in (0, 1)$ and $\eta_2 > \eta_2^*$.

If we do the intersection between the conditions of $D < 1$ and those of $T < 1 + D$ and considering that $\varepsilon_{11}^{**} < \varepsilon_{11}^D$ and $\eta_2^D < \eta_2^*$ for $\varepsilon_{11} < \varepsilon_{11}^{**}$, we get the local indeterminacy conditions summarized in Proposition (2). ■

References

[1] Becker, R. A. (1980), "On the Long-run Steady State in a Simple Dynamic Model of Equilibrium with Heterogeneous Households", Quarterly Journal of Economics 95, 375-382.

[2] Benhabib, J., and Farmer, R. (1994), "Indeterminacy and Increasing Returns", Journal of Economic Theory 63, 19-41.

[3] Bosi, S. and Seegmuller, T. (2008), "Can Heterogeneous Preferences Stabilize Endogenous Fluctuations?", Journal of Economic Dynamics and Control 32, 624-647.

[4] Bosi, S. and Seegmuller, T. (2010), "On the Role of Progressive Taxation in a Ramsey Model with Heterogeneous Households", Journal of Mathematical Economics 45, pp. 977-996.

[5] Cazzavillan, G., and Pintus, P. (2004), "Robustness of Multiple Equilibria in OLG Economies", Review of Economic Dynamics 7, 456-475.

[6] Cazzavillan, G., Llyod-Braga, T. and Pintus, P. (1998), "Multiple Steady States and Endogenous Fluctuations with Increasing Returns to Scale in Production", Journal of Economic Theory 80, 60-107.

[7] Chen, Y., and Zhang, Y. (2009a), "Endogenous Income Taxes in OLG Economies", MPRA Paper No. 16412.

[8] Chen, Y., and Zhang, Y. (2009b), "Endogenous Income Taxes in OLG Economies: A Clarification", MPRA Paper No. 16824.

[9] Dromel, N., and Pintus, P. (2006), "Are progressive Fiscal Rules Stabilizing?", Document de Travail 05, GREQAM.

[10] Dromel, N., and Pintus, P. (2007), "Linearly Progressive Income Taxes and Stabilization", Research in Economics 61, 28-29.

[11] Dromel, N., and Pintus, P. (2008), "Are Progressive Income Taxes Stabilizing?", Journal of Public Economic Theory 10, 329-349.

[12] Gokan, Y. (2009a), "Dynamic Effects of Government Budgetary policies in Reichlin's Overlapping Generations Model with Externalities", Mimeo.

[13] Guo, J.-T. (1999), "Multiple Equilibria and Progressive Taxation of Labor Income", Economic Letters 65, 97-103.

[14] Guo, J.-T., and Harrison, S. G. (2004), "Balanced-Budget Rules and Macroeconomic (In) stability", Journal of Economic Theory 119, 357-363.

[15] Guo, J.-T., and Harrison, S. G. (2008), "Useful Government Spending and Macroeconomic (In)stability Under Balanced-Budget Rules", Journal of Public Economic Theory 10, 383-397.

[16] Guo, J.-T., and Lansing, K. (1998), "Indeterminacy and Stabilization Policy", Journal of Economic Theory 82, 481-490.

[17] Hall, R. and Rabushka, A. (1995), "The Flat Tax", 2nd Edition, Hoover Institute Press, California.

[18] Llyod-Braga, T., Modesto, L., and Seegmuller T. (2008), "Tax Rate Variability and Public Spending As Sources of Indeterminacy", Journal of Public Economic Theory 10, 399-421.

[19] Musgrave, R.A., and Thin, T. (1948), "Income Tax Progression, 1929-1948", Journal of Political Economy 56, 498-514.

[20] Reichlin, P., (1986), "Equilibrium Cycles in an Overlapping Generations Economy with Production", Journal of Economic theory 40, 89-102.

[21] Sarte, P.-D. (1997), "Progressive Taxation and Income Inequality in Dynamic Competitive Equilibrium", Journal of Public Economics 66, 145-171.

[22] Seegmuller, T. (2003), "Endogenous Fluctuations and Public Services in a Simple OLG Economy", Economic Bulletin 5, No.10, 1-7.

[23] Schmitt-Grohé, S., and Uribe, M. (1997), "Balanced-Budget Rules, Distortionary Taxes, and Aggregate Instability", Journal of political Economy 108, No. 05.

[24] Sorger, G. (2002), "On the Long-run Distribution of Capital in the Ramsey Model", Journal of Economic Theory 105, 226-243.

[25] Woodford, M. (1986), "Stationary Sunspot Equilibria in a Finance Constraint Economy", Journal of Economic Theory 40, 128-137.