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Endogenous Timing in Two-Player Contests with Positive Spillovers

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Abstract

This paper examines simultaneous versus sequential choice of effort in a twoplayer contest with a logit type contest success function (CSF) in the presence of positive externalities. The timing of moves, determined in a pre-play stage prior to the contest-subgame is allowed to be endogenous. Contrary to endogenous timing models with no externalities the present paper finds that players may decide to choose their effort simultaneously in the subgame perfect equilibrium (SPE) of the extended game. Moreover, symmetry among players does not rule out incentives for precommitment to effort locally away from the Nash-Cournot level. However, the following conclusion still holds in a contest model with positive externalities: If the unique SPE is sequential play, the win probability in the NE is crucial for the determination of an endogenous leadership.

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Abstract

This paper examines simultaneous versus sequential choice of effort in a two-player contest with a logit type contest success function (CSF) in the presence of positive externalities. The timing of moves, determined in a pre-play stage prior to the contest-subgame is allowed to be endogenous. Contrary to endogenous timing models with no externalities the present paper finds that players may decide to choose their effort simultaneously in the subgame perfect equilibrium (SPE) of the extended game. Moreover, symmetry among players does not rule out incentives for precommitment to effort locally away from the Nash-Cournot level. However, the following conclusion still holds in a contest model with positive externalities: If the unique SPE is sequential play, the win probability in the NE is crucial for the determination of an endogenous leadership.

Keywords: Contests, Endogenous timing, Externalities
JEL classification: C72, D23, D30

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1 Introduction

This paper examines simultaneous versus sequential choice of effort in a two-player contest with a logit type contest success function (CSF) in the presence of positive externalities. The timing of moves, determined in a pre-play stage prior to the contest-subgame is allowed to be endogenous. Contrary to endogenous timing models with no externalities the present paper finds that players may decide to choose their effort simultaneously in the subgame perfect equilibrium (SPE) of the extended game. Moreover, symmetry among players does not rule out incentives for precommitment to effort locally away from the Nash-Cournot level. However, the following conclusion still holds in a contest model with positive externalities: If the unique SPE is sequential play, the win probability in the NE is crucial for the determination of an endogenous leadership.

2 The Modell

The exogenous prize of common value is given by R. The probability of winning is given by a specific logit type CSF.[1] Player i's probability of winning is given by

$$p^i(\mathbf{x}) = \begin{cases} \frac{\alpha_i\, x_i}{\alpha_i\, x_i + \alpha_j\, x_j} & \text{for } \mathbf{x} \neq 0, \\ \frac{1}{2} & \text{else.} \end{cases}$$

The resulting payoff function of player i is

$$\Pi^i\,(\mathbf{x}) = p^i(\mathbf{x})\,R - x_i + \kappa\, x_j. \tag{1}$$

Each agent maximizes his expected payoff which equals the prize that goes to the sole winner, weighted by the probability that he wins the contest minus the sure effort cost. Additionally, we assume that each player receives a benefit or loss which is proportional to the other player's effort.The best-response function of player i is thus given by

$$BR^i(x_j) \quad = \quad \begin{cases} \sqrt{R\,\theta_j\, x_j} - \theta_j\, x_j & \text{for } x_j < \frac{R}{\theta_j}, \\ 0 & \text{else,} \end{cases} \tag{2}$$

[1]This specific form has been used by Grossman (2001) and Hoffmann (2010).

where $\theta_j = \dfrac{\alpha_j}{\alpha_i}$ represents the relative efficiency of player j. Thus, as long as player j's effort is not too large player i's effort is strictly positive. In order to detect whether this is a game of PS or PC, and whether player i regards the effort of player j as SC or SS, we need to determine the cross derivatives of the payoff as well as of the marginal payoff function. Due to the properties of the payoff function itself, these cross derivatives depend in a non-monotonic way on the competitor's effort:

$$\Pi^i_j(\mathbf{x}) = p^i_j(\mathbf{x})R + \kappa \qquad \text{and} \qquad \Pi^i_{ij}(\mathbf{x}) = p^i_{ij}(\mathbf{x})R, \tag{3}$$

Thus, we will define the above concepts in the neighborhood of the NE.

2.1 Efforts in the three basic games

Solving simultaneously the FOCs for both players leads to the following unique NE-level of efforts

$$x^N_1 = x^N_2 = \frac{\theta_i\, R}{(1+\theta_i)^2}. \tag{4}$$

and therefore to a win probability of agent i in the NE of the game of

$$p^i(\mathbf{x}^N) = \frac{\theta_i}{1+\theta_i}. \tag{5}$$

The NE-Payoffs are then given by

$$\Pi^i(\mathbf{x}^N) = \frac{R\,\theta_i(\kappa+\theta_i)}{(1+\theta_i)^2}. \tag{6}$$

In order to guarantee $\Pi^i(x^N) > 0$ for all $i = 1,2$ we will assume that $\kappa > \underline{\kappa} \equiv \max\{-\theta_1, -\theta_2\}$. Given eq. (4) we find that

$$\Pi^i_j(\mathbf{x^N}) = -1+\kappa \quad \text{and} \quad \Pi^i_{ij}(\mathbf{x^N}) = \frac{\theta_i^2-1}{R\,\theta_i}. \tag{7}$$

Hence, whether we have a game of PC or PS solely depends on the value of the externality factor (κ), i.e. if $\kappa > 1$ ($\kappa < 1$) we have a game of PC (PS). The strategic incentives, however, depend solely on the relative effectivity parameter (θ_i). Due to the symmetry of the CSF we know that

$$p^i_{ij}(\mathbf{x}) = \frac{\theta_i^2-1}{R^2\theta_i} = -p^j_{ij}(\mathbf{x}), \tag{8}$$

so that $\Pi^i_{12}(\mathbf{x}^N) = -\Pi^j_{12}(\mathbf{x}^N) = p^i_{12}(\mathbf{x}^N)\,R$. Accordingly, either $\theta_i = 1$ and the strategic incentives are aligned and equal to zero, or $\theta_i \neq 0$ and the strategic incentives are directly opposed. Since

$$\Pi^i_{ij}(\mathbf{x}^N) \left\{ \begin{matrix} > \\ = \\ < \end{matrix} \right\} 0 \Leftrightarrow \theta_i \left\{ \begin{matrix} > \\ = \\ < \end{matrix} \right\} 1 \Leftrightarrow p^i\left(\mathbf{x}^N\right) \left\{ \begin{matrix} > \\ = \\ < \end{matrix} \right\} \frac{1}{2}. \tag{9}$$

we find that the favorite's (underdog's) effort is SS (SC) to the underdog's (favorite's) effort, as stated by Dixit (1987).

These facts are represented in figure 1, where $\theta_1 > 1$ and $\kappa > 1$. The bold lines represent the best response functions of the two players. Apparently, the strategy

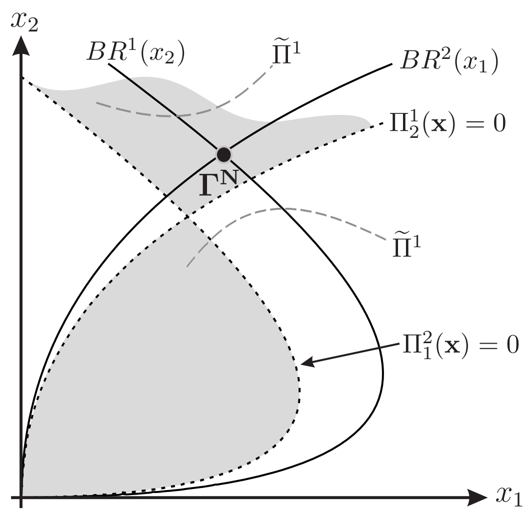

Figure 1
4 different regimes

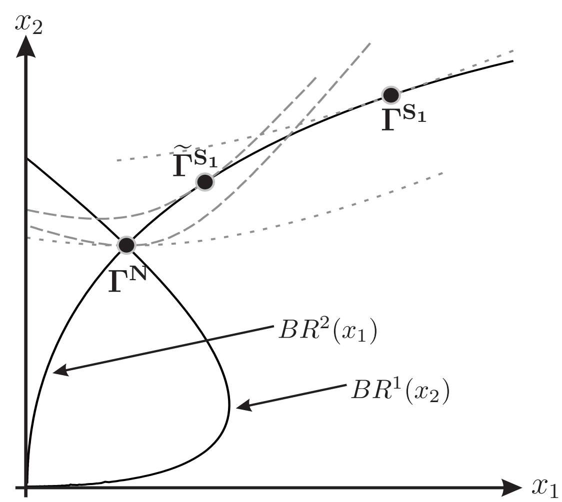

Figure 2
Different positive externalities

of player 2 (1) is a SC (SS) to that of player 1 (2), since $\theta_1 > 1$. The dotted lines represent the strategy profiles for which $\Pi^i_j(\mathbf{x}) = 0$, with $i \neq j$, i.e. those strategy profiles for which the net-effect of an increase in the opponents strategy on the payoff is exactly zero. Below (above) this dotted line $\Pi^i_j(\mathbf{x}) < 0$ ($\Pi^i_j(\mathbf{x}) > 0$). Since the NE lies above both dotted lines, we know that $\Pi^i_j(\mathbf{x}^N) > 0$ and, given equation (6), that $\kappa > 1$. Note that for example $\Pi^1(\mathbf{x}) = \widetilde{\Pi}^1$ can be represented by two different strategy profiles, one for which $\Pi^1_2(\mathbf{x}) > 0$ (the convex iso-payoff-curve) and one for $\Pi^1_2(\mathbf{x}) < 0$ (the concave iso-payoff-curve).

Next, we turn to the sequential move games. The subgame perfect equilibrium of the contest subgame (the Stackelberg equilibrium) is determined by applying backward induction. In the game where agent j leads (Γ^{S_j}) the follower's optimal behavior is given by eq. (2). The leader's maximization program is then given by

$$x_j^L \equiv \underset{x_j}{\text{argmax}}\ \Pi^j(BR^i(x_j), x_j)$$

which yields the following level of efforts in the SPE of the game:

$$x_i^F = \begin{cases} 0, \\ \dfrac{R(1+\kappa)\theta(2+(1-\kappa)\theta)}{4(1+\kappa\,\theta)} \end{cases} \quad \text{and} \quad x_j^L = \begin{cases} R\theta_j & \text{for } \kappa \in \mathcal{Z}, \\ \dfrac{R\,(1+\kappa)^2\theta_j}{4(1+\kappa\,\theta_j)^2} & \text{else,} \end{cases} \tag{10}$$

with $\mathcal{Z} = \left(\underline{\kappa}, \frac{\theta_j - 2}{\theta_j}\right)$, which is only non-empty for $\theta_j > 1$. Hence, a leader-underdog ($\theta_j < 1$) can never induce a follower-favourite effort of $x_i^F = 0$. However, in the case where the leader is the favourite ($\theta_j > 1$), a follower-underdog effort of $x_i^F = 0$ may emerge in equilibrium, even if externalities are positive. For this to emerge the externalities have to be sufficiently small or negative, i.e. $\kappa < \frac{\theta_j - 2}{\theta_j}$. Comparing our findings between the NE and two Stackelberg equilibria leads to the following lemma.

Lemma 1

The sign of the difference in the follower's effort compared to his effort in the NE does only depend on the value of the externality parameter, as long as $\Pi^j_{ij}(\mathbf{x}^N) \neq 0$. *In particular we find that*

$$x_i^F \left\{ \begin{matrix} > \\ = \\ < \end{matrix} \right\} x_i^N \Leftrightarrow \Pi^i_j(\mathbf{x}^N) \times \left|\Pi^i_{ij}\left(\mathbf{x}^N\right)\right| \left\{ \begin{matrix} > \\ = \\ < \end{matrix} \right\} 0.$$

However, the difference in the leader's effort compared to his NE-effort depends on the value of the externality as well as on the strategic incentives, in particular

$$x_j^L \left\{ \begin{matrix} > \\ = \\ < \end{matrix} \right\} x_j^N \Leftrightarrow \Pi^j_i(\mathbf{x}^N) \times \Pi^j_{ij}(\mathbf{x}^N) \left\{ \begin{matrix} > \\ = \\ < \end{matrix} \right\} 0.$$

Suppose that we have a game of PC ($\kappa > 1$). Hence, due to the positive net effect of any marginal increase in x_i on the leader's payoff, the leader will always try to increase the follower's effort compared to the NE-value. A leader-favourite (leader-underdog) will thus decrease (increase) his effort compared to the NE, since his effort is a SS (SC) to the follower-underdog's effort. If either $\Pi^i_j(\mathbf{x}^N) = 0$ (i.e., $\kappa = 1$) or $\Pi^i_{ij}(\mathbf{x}^N) = 0$ (i.e. $\theta_i = 1$) the leader's payoff is maximized at $x_j^L = x_j^N$. Partly, this results follows Dixit (1987), who stated that in the case of symmetry ($\theta_i = 1$) no leader has an incentive to deviate from his NE-value of x_j.[2] In our setting this also true if the externality parameter $\kappa = 1$. In this case a marginal increase of the follower's effort has no impact on the leader's payoff.

Moreover, the difference in the leader-effort an the NE-effort increases in the following difference: $\kappa - 1$, i.e. the stronger κ deviates from the zero-case, the stronger will be the difference, measured in effort, between Γ^{S_i} and Γ^N for player i. The same holds for .....

The corresponding payoffs of the Stackelberg-follower, and, respectively, -leader are

$$\Pi^i\left(\mathbf{x}^{S_j}\right) = \begin{cases} \kappa\,\theta_i\,R, & \text{for } \kappa \in \mathcal{Z}, \\ \dfrac{R(4+\theta(\theta+\kappa(5-2\theta+\alpha(2+\alpha+\theta))-4))}{4(1+\kappa\,\theta)^2} & \text{else,} \end{cases} \tag{11a}$$

and

$$\Pi^j\left(\mathbf{x}^{S_j}\right) = \begin{cases} R(1-\theta_i) & \text{for } \kappa \in \mathcal{Z}, \\ \dfrac{R\,(1+\kappa)^2\theta_j}{4(1+\kappa\,\theta_j)} & \text{else,} \end{cases} \tag{11b}$$

Given these rankings, we can now compare the payoffs in the three basic games (Γ^N, Γ^{S_1} and Γ^{S_2}), which will give us the opportunity of detecting potential first-mover (second-mover) advantages or first-mover (second-mover) incentives, which we need for our last two lemmas. We define these concepts as follows:

Definition 1 (First-mover (second-mover) advantage & second-mover (dis-)incentive) *Player i has a first-mover advantage (FMA) if his leader-payoff exceeds his follower-payoff. The opposite holds if player i has a second-mover advantage (SMA). More formally*

$$\textit{Player } i \textit{ has a } \left\{ \begin{array}{c} \textit{first-mover advantage} \\ \textit{second-mover advantage} \end{array} \right\} \Leftrightarrow \Pi^i(\mathbf{x}^{S_i}) \left\{ \begin{array}{c} > \\ < \end{array} \right\} \Pi^i(\mathbf{x}^{S_j}).$$

Player i has a second-mover incentive (SMI) if his follower payoff exceeds or equal to his NE-Payoff. If the opposite holds player i has a second-mover disincentive

[2]Since the leader-payoff is strictly concave in his own strategy, we can rule out the case of an inflection point of the Stackelberg-leader payoff. See Baik et al. (1999).

(SMD):

$$\text{Player } i \text{ has a } \left\{ \begin{array}{l} \text{second-mover incentive} \\ \text{second-mover dis-incentive} \end{array} \right\} \Leftrightarrow \Pi^i(\mathbf{x}^{S_j}) \left\{ \begin{array}{c} \geq \\ < \end{array} \right\} \Pi^i(\mathbf{x}^N).$$

We will now turn to our second lemma:

Lemma 2 (SMI and SMD)
Player i has a SMI if

1. *the game is symmetric* ($\theta_j = 1$),
2. $\Pi^i_j(\mathbf{x}^N) = 0$ *for* $i \neq j$ *(i.e.* $\kappa = 1$*),*
3. *the Stackelberg-leader is an underdog* ($\theta_j < 1$),
4. *the Stackelberg-leader is a favourite* ($\theta_j > 1$) *and either*

    a. $\kappa \in \mathbb{A} = \left\{ \kappa, \theta_j \middle| \frac{\theta_j}{1+2\theta_j} < \kappa < \frac{\theta_j - 2}{\theta_j} \right\}$ *or*

    b. $\kappa \in \mathbb{B} = \left\{ \kappa, \theta_j \middle| \frac{\theta_j - 2}{\theta_j} < \kappa < \frac{(\theta_j - 1)\theta_j - 4}{1+3\theta_j} \right\}$

The first and second part of lemma 2 stems from the fact that in these cases $x_i^N = x_i^L$ and $x_j^L = x_j^N$ (see lemma 1). The third part of lemma 2 is in line with the results of **?** and Leininger (1993), i.e. a follower-favourite always has a SMI, whereas the last part of the previous lemma only emerges in the presence of externalities. If the

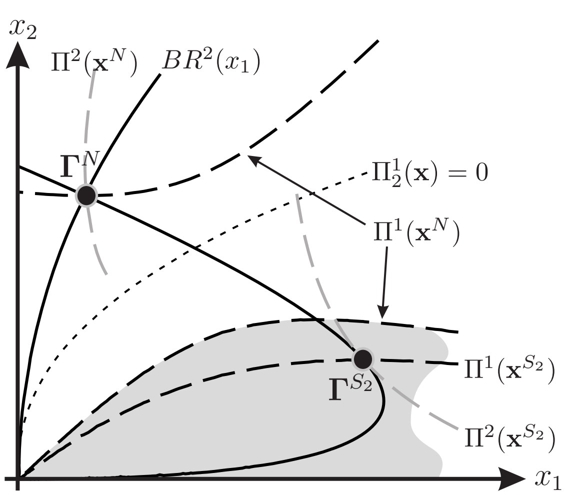

Figure 3
A SMI of an underdog

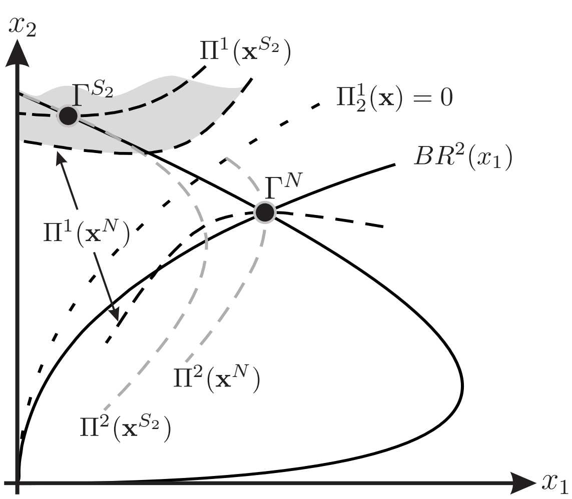

Figure 4
A SMI of an underdog

follower (player i) is an underdog and $\kappa \in \mathcal{A}$ the effort the Stackelberg-follower will be strictly positive, if $\kappa \in \mathcal{B}$ it will be zero. Nonetheless, in both cases player i has a SMI, i.e. $\Pi^i(x^{S_j}) > \Pi^i(x^N)$. A necessary condition for this to emerge is that the leader is sufficiently relatively effective, i.e. $\theta_j > \tilde{\theta}_j \equiv \frac{3+\sqrt{17}}{2}$, otherwise $\mathcal{A} \cup \mathcal{B}$ would be empty. The more relatively effective the leader, the larger will be the deviation of the follower-effort compared to the NE-level. If this deviation is large enough, then the follower-underdog might even be better off compared to the NE. Figure

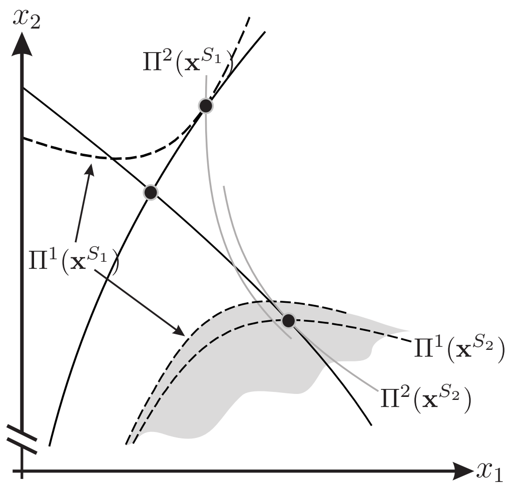

Figure 5
A SMI of an underdog

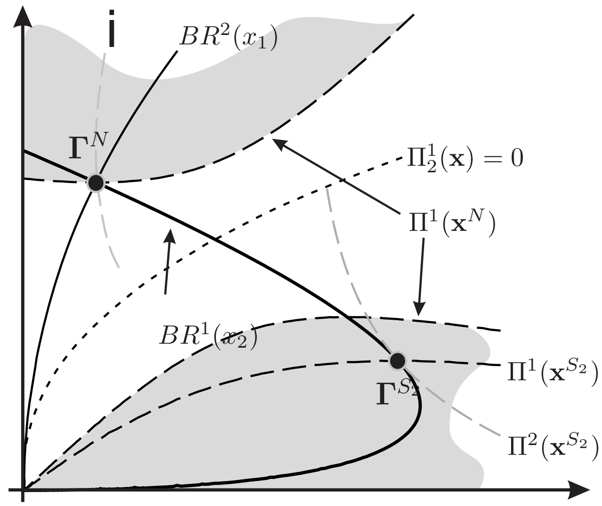

Figure 6
A SMI of an underdog

3 represents an example with $\kappa \in \mathcal{A}$ and $\kappa > 1$. Due to the latter assumption we have a game of PC for both players ($\Pi^i_j(\mathbf{x}^N) > 0$) and their corresponding iso-payoff curves ($\Pi^i(\mathbf{x}) = \Pi^i(\mathbf{x}^N)$) are strictly concave.

An equivalent iso-payoff curve for player 1 can be found for which $\Pi^i_j(\mathbf{x}) < 0$. Hence, the grey surface to the south of this concave payoff-curve represents all those strategy profiles which dominate the NE-payoff of player 1. Now suppose that the favourite (player 2) becomes a Stackelberg-leader. Given lemma 1 we know that the leader-favourite will decrease his effort in order to increase player 1's. The corresponding Stackelberg equilibrium (Γ^{S_2}) can thus be found to the south-east of Γ^N. Here, not only the leader is better off compared to the NE, the follower as well, since the iso-payoff curve of the follower (player 1) lies inside the grey surface, therefore showing that $\Pi^1(\mathbf{x}^{S_2}) > \Pi^1(\mathbf{x}^N)$. Figure 4 represents a case where $\kappa \in \mathcal{A}$ and $\kappa < 1$. Due to the latter assumption we now have a game of PS for both players ($\Pi^i_j(\mathbf{x}^N) < 0$) and their corresponding iso-payoff curves ($\Pi^i(\mathbf{x}) = \Pi^i(\mathbf{x}^N)$) are strictly convex.

An equivalent iso-payoff curve for player 1 can be found for which $\Pi^i_j(\mathbf{x}) > 0$. Hence, the grey surface to the north of this convex payoff-curve represents all those strategy

profiles which dominate the NE-payoff of player 1. Now suppose that the leader-favourite (player 2) will increase his effort in order to decrease player 1's. The corresponding Stackelberg equilibrium (Γ^{S_2}) can thus be found to the north-west of Γ^N. Here, not only the leader is better off compared to the NE, the follower as well, since the iso-payoff curve of the follower (player 1) lies inside the grey surface, therefore showing that $\Pi^1(\mathbf{x}^{S_2}) > \Pi^1(\mathbf{x}^N)$.
Next lemma.

Lemma 3
Player i has a SMA and player j has a FMA

1. *if player j is an underdog ($\theta_j < 1$) and $\kappa \in \mathcal{C}$,*
2. *if player j is a favourite ($\theta_j > 1$) and $\kappa \notin \mathcal{D}$*

Figure 3 represents an example with $\kappa \in \mathcal{A}$ and $\kappa > 1$. Due to the latter assumption we have a game of PC for both players ($\Pi^i_j(\mathbf{x}^N) > 0$) and their corresponding iso-payoff curves ($\Pi^i(\mathbf{x}) = \Pi^i(\mathbf{x}^N)$) are strictly concave.

3 Conclusion

Based on the endogenous timing game by Hamilton and Slutsky (1990), we have provided a framework for the analysis of endogenous leadership in contests with positive externalities. In a stage prior to the contest, the players decided whether they will exert effort as soon as or as late as possible; and their decision, to which they are committed, is announced to the other player subsequently. In this model we have provided a taxonomy of endogenous leadership. We were able to generalize the findings of Baik and Shogren (1992) and Leininger (1993) regarding the behavior of the Stackelberg-leader as well as the fact that the SPE of the extended game is always Pareto-undominated. However, there are differences compared to the aforementioned literature. In particular, we were able to establish that the SPE of the extended game may be represented by a simultaneous move game.

Our work can be extended in various ways:

Regarding the previous work of Yildirim (2005) and Romano and Yildirim (2005) it would be interesting to establish in which way the findings of the present paper would be modified if one abstains from the assumption that each player is allowed to exert effort only once. For instance, in the case were players are evenly matched, Yildirim (2005) finds that the outcome of the game is equivalent to a game where players move simultaneously, although effort might be exerted *early* and *late*. Therefore, allowing the players in our framework to exert effort twice might eliminate the coordination issue in a game of strategic complements.

This extensions are the subject of current research.

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