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Smooth politicians and altruistic voters: a new approach to modeling large elections

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Abstract

We propose a new approach to modeling large elections that overcomes the "paradox of voting" in a costly voting framework, without reliance on the assumption of direct psychic rewards from casting one's ballot. We consider a two-party system and modify the standard model in two ways. First, we drop the usual winner-take-all assumption and instead include a "smooth" policy rule under which the degree to which parties favor their own interests is increasing in their margin of victory. Second, we assume that voters are altruistic. Citizens not only get a private benefit as a function of the proportion of votes cast for the party they support, but they also receive spillovers from the impact that the policy has on other individuals. We consider two types of altruistic citizens. Group Minded voters only receive spillovers from the benefit enjoyed by supporters of their own party. Paternalistic voters also receive spillovers from imposing the policy of their preferred alternative on the supporters of the other party. When the size of the electorate grows without bound, limiting turnout rate is strictly positive if and only if the supporters of both parties are altruistic. Our model generates comparative static predictions that in accordance with most of the standard costly voting literature: the closer the election the higher turnout is predicted (competition effect); turnout rate is higher for the minority party (underdog effect).

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Smooth Politicians and Altruistic Voters:
A New Approach to Modeling Large Elections[1]

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Abstract

We propose a new approach to modeling large elections that overcomes the "paradox of voting" in a costly voting framework, without reliance on the assumption of direct psychic rewards from casting one's ballot. We assume that voters are altruistic, and we include a "smooth" policy rule under which the degree to which parties favor their own interests is increasing in their margin of victory. We show that, when the size of the electorate grows without bound, limiting turnout rate is strictly positive.

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Smooth Politicians and Altruistic Voters:
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I. Introduction

Since the seminal work of Palfrey and Rosenthal (1983), economists have commonly modeled elections as participation games where voters pay a cost to vote (e.g. Palfrey and Rosenthal, 1985; Campbell, 1999; Börgers, 2004; Goeree and Grosser, 2007; Krasa and Polborn, 2009; Krishna and Morgan, 2010; Taylor and Yildirim 2011, among others). The popularity of such models can be explained by their game theoretic micro foundations and their ability to generate predictions which are consistent with evidence of voters' strategic behavior (e.g. Riker and Ordeshook, 1968; Franklin et al., 1994), and comparative static results that are supported by the majority of empirical studies (e.g. Shachar and Nalebuff, 1999; Blais, 2000). While popular, current formulations of the costly voting model confront one major drawback – the "paradox of voting" which was first described in decision theoretic terms by Anthony Downs (1957). To date, the prediction that turnout rate converges to zero as the size of the electorate grows has only been overcome in the costly voting model through the incorporation of ad hoc preferences for voting (see for instance Harsanyi, 1977, 1992; Feddersen and Sandroni, 2006; Coate and Conlin, 2004). We propose a new approach to modeling large elections that overcomes this paradox in a costly voting framework, without reliance on the assumption of direct psychic rewards from casting one's ballot.

We consider a two-party system. Citizens are characterized by their political preference and cost to voting, both being private information, and decide whether to vote or abstain. We modify the standard model in two ways. First, we drop the usual winner-take-all assumption and instead include a "smooth" policy rule under which the degree to which parties (or elected officials) favor their own party's interests is increasing in their margin of victory. Specifically, we assume that the benefit from government action is distributed according to a continuous function that is strictly increasing in the proportion of votes received by a given party. Politicians are then “smooth” in the sense that a large majority is preferred to a thinner one, even when they receive more than fifty per cent of the votes.

Second, we assume that voters are altruistic. Citizens not only get a private benefit as a function of the proportion of votes cast for the party they support, but they also receive spillovers from the impact that the policy has on other individuals. We consider two types of altruistic citizens. Group Minded voters only receive spillovers from the benefit enjoyed by supporters of their own party. Paternalistic voters also receive spillovers from imposing the policy of their preferred alternative on the supporters of the other party.[4]

To motivate our assumption about “smooth” politicians, we provide clear empirical evidence of this type of behavior in the U.S. Congress. In particular, we use a panel data model with member and Congress fixed effects to demonstrate that the degree to which members of congress adopt partisan voting records is increasing in the margin of victory in their most recent election.

Altruistic voting behavior is more difficult to identify directly in the data. Here, we are motivated by the large theoretical literature and experimental evidence regarding altruistic behavior (see for instance Becker, 1976; Hirshleifer, 1977; Margolis, 1982; Eckel and Grossman, 1996; Andreoni and Vesterlund, 2001; Andreoni and Miller, 2002)**.** Also, following on arguments made in the Political Science literature (see for instance Edlin et. al. 2007) we highlight that altruism is one (of many possible) explanations for why turn out in the United States is highest in presidential elections and higher in gubernatorial elections than in mayoral elections.

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[4] Technically, Group Mindedness and Paternalism are both forms of paternalistic altruism, in that citizens benefit from imposing their preferred policy on other individuals, whether they share or not their political preference.

We show that, when the size of the electorate grows without bound, limiting turnout rate is strictly positive if and only if the supporters of both parties are altruistic. The intuition for this result is that, although in the limit the weight of each vote becomes negligible, the benefit from voting increases as the population grows. When the supporters of the two parties display the same overall level of altruism, our model generates comparative static predictions that in accordance with most of the standard costly voting literature. Specifically, the closer the election the higher turnout is predicted (*competition effect*). Moreover, if voters are Paternalistic, in equilibrium members of the minority party turn out to vote relatively more frequently than the supporters of the other alternative (*underdog effect*). Both these predictions are supported by empirical studies (see Shachar and Nalebuff, 1999; Blais, 2000). Finally, we provide sufficient conditions for the equilibrium to be unique.

II. The Model

We consider a model of costly voting with two parties: $P = A, B$. Society is composed of $N + 1$ citizens. Each individual has got the same ex ante independent probability $\lambda \in (0,1)$ of being a supporter of party A and $1 - \lambda$ of supporting party B. Citizens decide simultaneously whether to vote or to abstain. If they decide to participate in the election they bear a cost. We assume that for a generic individual i, supporter of party P, there is a cost to voting $c_{i \in P} \in \left[\underline{c}_P, \overline{c}_P\right] \subset \mathbb{R}_+$. Members of party P draw their voting costs independently from the differentiable distribution $F_P(c_P)$, with $F_P{}'(c_P) > 0$ on all the support. While λ, F_A and F_B are common knowledge, each citizen's preference and cost to voting are private information. If at least one individual votes, then each member of party P receives a benefit from government action according to the electoral rule $G: [0,1] \rightarrow [0,1]$, which is a function of the proportion of total votes z_P obtained by party P. We assume $G'(z_P) > 0$ and finite, $G(0) = 0$, $G(1) = 1$ and $G(z_P) = 1 - G(1 -$

z_P). If no one votes then the benefit from government action received by each individual is equal to $\frac{1}{2}$.

Individuals receive direct benefits from government allocations, but they also exhibit altruism. They receive spillovers from the benefits obtained by the other members of their party and may enjoy positive utility from each member of the other party being subject to their own party's policy. The latter may be interpreted as paternalistic altruism. Alternatively, it could also be interpreted as a form of spiteful preference. For example, we could think of it as the utility that a supporter of a party enjoys from imposing her own preferred policy on her political adversaries. Call $\gamma_P^P \geq 0$ the weight that a member of party P places on the welfare of each other supporter of her party, while $\gamma_P^{\bar{P}} \geq 0$ represents the utility she receives from imposing party P's policy on a member of the opposite party. We assume that γ_P^P and $\gamma_P^{\bar{P}}$ are common knowledge.

Consider individual i, with cost c_i, belonging to party P. Call N_P and V_P the number of supporters of party P and of votes cast for P, both exclusive of individual i, respectively. Moreover, define by $V_{\bar{P}}$ the number of votes cast for the other party. Notice that for a supporter of party P voting for the other party is dominated by abstaining, hence citizens' actions boil down to abstain or to vote for their preferred alternative. Then i's net benefit of voting is given by

$$u_{i\in P} = \begin{cases} \left[1 + N_P\gamma_P^P + (N - N_P)\gamma_P^{\bar{P}}\right]\left[G\left(\dfrac{V_P + 1}{V_P + V_{\bar{P}} + 1}\right) - G\left(\dfrac{V_P}{V_P + V_{\bar{P}}}\right)\right] - c_i & if\ V_P + V_{\bar{P}} > 0 \\ \dfrac{1}{2}\left[1 + N_P\gamma_P^P + (N - N_P)\gamma_P^{\bar{P}}\right] - c_i & if\ V_P + V_{\bar{P}} = 0 \end{cases}$$

The solution concept that we employ is Bayesian-Nash equilibrium (BNE). As it is customary in this literature, we restrict our attention to type-symmetric Bayesian-Nash equilibria, in the sense that all citizens supporting the same alternative choose the same strategy. In turn, participation decisions depend on the realization of the individual voting cost. Formally, a strategy is a mapping $s_P\colon [\underline{c}_P, \overline{c}_P] \to \{0,1\}$, where $s_P(c_i) = 0$ means that individual i supporting party P abstains and votes otherwise. A strategy profile $\{s_A, s_B\}$ is a type-symmetric BNE of the game if $s_P(c_i)$ maximizes every individual's expected payoff, given that all other individuals adhere to s_P.

We start by exploring voters' behavior when $N+1$ is finite. It is possible to characterize citizens' strategies through cut-off values c_P^* such that

$$s_P(c_i) = \begin{cases} 1 & if\, c_i \le c_P^* \\ 0 & if\, c_i > c_P^* \end{cases} \tag{1}$$

Proposition 1 *There exists a pure strategy type-symmetric BNE characterized by the voting strategy in (1) and thresholds c_A^* and c_B^*.*

III. Equilibrium in Large Elections

Having characterized the equilibrium for $N+1$ finite, we now turn to analyze voters' behavior in large elections. In order to do this we need to introduce some extra notation. First of all, call $\nu_A \ge 0$ the probability that an A-supporter votes in the election. Equally, $\nu_B \ge 0$ is the probability that a member of party B votes. Moreover, given individual i, call $\hat{\lambda}$ and $(1-\hat{\lambda})$ the realized proportions of the electorate, exclusive of i, that support party A and B, respectively. Finally, call $\hat{\nu}_A$ and $\hat{\nu}_B$ the realized proportions of the electorate, exclusive of i, that vote for party A and B, respectively.

For simplicity of notation let us define $\Gamma_A = \gamma_A^A \lambda + \gamma_A^B (1-\lambda)$ and $\hat{\Gamma}_A = \gamma_A^A \hat{\lambda} + \gamma_A^B (1-\hat{\lambda})$. Similarly, for a B-supporter, $\Gamma_B = \gamma_B^A \lambda + \gamma_B^B (1-\lambda)$ and $\hat{\Gamma}_B = \gamma_B^A \hat{\lambda} + \gamma_B^B (1-\hat{\lambda})$.

With no loss of generality, consider a supporter of party A. Given a sample of size $N+1$, conditional on the decisions of all other individuals, the expected benefit from voting for an A-

supporter is given by Equation (2).

$$E[B_A] = E\left[\left(1+N\hat{\Gamma}_A\right)\left[G\left(\frac{\hat{v}_A\hat{\lambda}N+1}{\hat{v}_A\hat{\lambda}N+\hat{v}_B(1-\hat{\lambda})N+1}\right)-G\left(\frac{\hat{v}_A\hat{\lambda}}{\hat{v}_A\hat{\lambda}+\hat{v}_B(1-\hat{\lambda})}\right)\right]\middle|\,\hat{v}_A\hat{\lambda}N+\hat{v}_B\left(1-\hat{\lambda}\right)N>0\right]* P\left[\hat{v}_A\hat{\lambda}N+\hat{v}_B\left(1-\hat{\lambda}\right)N>0\right]+E\left[\frac{1}{2}\left(1+N\hat{\Gamma}_A\right)\middle|\hat{v}_A\hat{\lambda}N+\hat{v}_B\left(1-\hat{\lambda}\right)N=0\right]*P\left[\hat{v}_A\hat{\lambda}N+\hat{v}_B\left(1-\hat{\lambda}\right)N=0\right] \quad (2)$$

The following proposition summarizes the main result of the paper.

Proposition 2 *When $N \to \infty$, limiting turnout is positive if and only if $\Gamma_A > 0$ and $\Gamma_B > 0$. If $\Gamma_A > 0$ and $\Gamma_B > 0$ the expected gross benefit from voting for an A-member is given by $\Gamma_A \frac{F_B(c_B^*)(1-\lambda)}{\left[F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)\right]^2} G'\left(\frac{F_A(c_A^*)\lambda}{F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)}\right)$, while it is equal to $\Gamma_B \frac{F_A(c_A^*)\lambda}{\left[F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)\right]^2} G'\left(\frac{F_B(c_B^*)(1-\lambda)}{F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)}\right)$ for a B-member. If $\Gamma_A = \Gamma_B = 0$, $\underline{c}_A \geq \frac{1}{2}$ and $\underline{c}_B \geq \frac{1}{2}$ then turnout is equal to zero, while it converges to zero otherwise.*

The next corollary follows immediately from the proof of the previous proposition.

Corollary 1 *Given $G(.)$ and λ, there exists a pair $(\tilde{\Gamma}_A, \tilde{\Gamma}_B)$ such that $c_A^* = \overline{c}_A$ and $c_B^* = \overline{c}_B$ for any (Γ_A, Γ_B) with $\Gamma_A \geq \tilde{\Gamma}_A$ and $\Gamma_B \geq \tilde{\Gamma}_B$. Given $G(.)$, λ and $\Gamma_{\overline{P}} > 0$ there exists $\check{\Gamma}_P$ such that $c_P^* = \overline{c}_P$ $\forall$ $\Gamma_P \geq \check{\Gamma}_P$.*

We focus on large elections where limiting turnout is positive, that is $\Gamma_{\mathrm{A}} > 0$ and $\Gamma_{\mathrm{B}} > 0$. We consider two different types of voters. If members of party P are such that $\gamma_P^{\overline{P}} = 0$ we will say that they are Group Minded, as they only care about the members of their own party. On the other hand, we will call them Paternalistic if $\gamma_P^P > 0$ and $\gamma_P^{\overline{P}} > 0$. We limit our analysis to the cases where all citizens are either Group Minded or Paternalistic. Most results carry through to the case where the supporters of one party are Group Minded, while the members of the opposite party are Paternalistic. However, the study of the "mixed" case would distract the reader from the main results, without adding any interesting insights.[5]

Lemma 1 $\frac{\partial(c_A^* F_A(c_A^*)/c_B^* F_B(c_B^*))}{\partial\gamma_A^P} > 0$ *and* $\frac{\partial(c_A^* F_A(c_A^*)/c_B^* F_B(c_B^*))}{\partial\gamma_B^P} < 0$. *It cannot be the case that* $\frac{\partial c_A^*}{\partial\gamma_A^P} < 0$ *and* $\frac{\partial c_B^*}{\partial\gamma_A^P} > 0$*; similarly, it cannot be that* $\frac{\partial c_A^*}{\partial\gamma_B^P} > 0$ *and* $\frac{\partial c_B^*}{\partial\gamma_B^P} < 0$.

Lemma 2

If voters are Paternalistic then $\frac{\partial(c_A^* F_A(c_A^*)/c_B^* F_B(c_B^*))}{\partial\lambda} < 0$*; it cannot be the case that* $\frac{\partial c_A^*}{\partial\lambda} > 0$ *and* $\frac{\partial c_B^*}{\partial\lambda} < 0$. *If voters are Group Minded then* $\frac{\partial(c_A^* F_A(c_A^*)/c_B^* F_B(c_B^*))}{\partial\lambda} = 0$ *and* $\frac{\partial c_A^*}{\partial\lambda} = \frac{\partial c_B^*}{\partial\lambda}$.

Proposition 3

i. *If voters are Paternalistic then* $\lim_{\lambda\to 0} F_A(c_A^*) = 1$ *and* $\lim_{\lambda\to 0} F_B(c_B^*) = 0$*, while* $\lim_{\lambda\to 1} F_A(c_A^*) = 0$ *and* $\lim_{\lambda\to 1} F_B(c_B^*) = 1$.

ii. *If voters are Group Minded then* $\frac{c_A^* F_A(c_A^*)}{c_B^* F_B(c_B^*)} = \frac{\gamma_A^A}{\gamma_B^B}$, $\lim_{\lambda\to 0} F_A(c_A^*) = 0$, $\lim_{\lambda\to 0} F_B(c_B^*) = 0$, $\lim_{\lambda\to 1} F_A(c_A^*) = 0$ *and* $\lim_{\lambda\to 1} F_B(c_B^*) = 0$*;*

Proposition 4 *Assume* $F_A = F_B = F$ *and suppose that voters are Paternalistic. There exists* $\hat{\lambda}$ *such that* $c_A^*(\lambda) > c_B^*(\lambda)\forall\lambda < \hat{\lambda}$, $c_A^*(\lambda) < c_B^*(\lambda)\forall\lambda > \tilde{\lambda}$ *and* $c_A^*(\tilde{\lambda}) = c_B^*(\tilde{\lambda})$.

i. *If* $\gamma_A^A + \gamma_A^B = \gamma_B^A + \gamma_B^B$ *then* $\tilde{\lambda} = \frac{1}{2}$ *(Underdog Effect).*

ii. *If* $\gamma_A^A + \gamma_A^B > \gamma_B^A + \gamma_B^B$ *then* $\tilde{\lambda} > \frac{1}{2}$.

iii. *If* $\gamma_A^A + \gamma_A^B < \gamma_B^A + \gamma_B^B$ *then* $\tilde{\lambda} < \frac{1}{2}$.

Proposition 5 *Assume* $F_A = F_B = F$ *and suppose that voters are Group Minded.*

i. *If* $\gamma_A^A = \gamma_B^B$ *then* $c_A^*(\lambda) = c_B^*(\lambda)\forall\lambda$.

ii. *If* $\gamma_A^A > \gamma_B^B$ *then* $c_A^*(\lambda) > c_B^*(\lambda)\forall\lambda$.

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[5] Similarly, provided some extra conditions, the main results would also hold in the case where $\gamma_P^{\bar{P}} > 0$, but $\gamma_P^P = 0$, that is when voters only care about the supporters of the opposite party. Nevertheless, we do not analyze this case as it is not realistic.

iii. *If* $\gamma_A^A < \gamma_B^B$ *then* $c_A^*(\lambda) < c_B^*(\lambda) \forall \lambda$.

Define θ_A^* as $\frac{\lambda F_A(c_A^*)}{\lambda F_A(c_A^*)+(1-\lambda)F_B(c_B^*)}$.

Proposition 6 *Assume* $F_A = F_B = F$ *and suppose that* $\gamma_A^A + \gamma_A^B = \gamma_B^A + \gamma_B^B$. *If* $\gamma_A^A > \gamma_B^A$ *and* $\gamma_B^B > \gamma_A^B$ *then* $\theta_A^* < \frac{1}{2}\ \forall\ \lambda < \frac{1}{2}$, *while* $\theta_A^* > \frac{1}{2}\ \forall\ \lambda > \frac{1}{2}$.

Proposition 7 (Competition Effect) *Assume* $F_A = F_B = F$ *and suppose that* $\gamma_A^A + \gamma_A^B = \gamma_B^A + \gamma_B^B$. *If* $G''(z_P) > 0$ *for* $z_P < \frac{1}{2}$ *then turnout is higher the closer* λ *is to* $\frac{1}{2}$.

Proposition 8 *Assume* $F_A = F_B = F$ *and* $G''(z_P) \geq 0$ *for* $z_P < \frac{1}{2}$. *Suppose also that* $\gamma_A^A + \gamma_A^B = \gamma_B^A + \gamma_B^B$, $\gamma_A^A > \gamma_B^A$ *and* $\gamma_B^B > \gamma_A^B$. *If* $\frac{F''}{F'} < \frac{F'}{F}$ *then the equilibrium is unique.*

Appendix

Proof of Proposition 1

Suppose citizens play according to the strategy defined by (1) and thresholds $\hat{c}_A$ and $\hat{c}_B$. Then for a supporter of party A the expected gross benefit from voting is given by

$$E[B_A(\hat{c}_A,\hat{c}_B)] = \sum_{N_A=0}^{N} \binom{N}{N_A} \lambda_A^{N_A}(1-\lambda_A)^{(N-N_A)} \sum_{V_A=0}^{N_A} \sum_{V_B=0}^{N-N_A} \binom{N_A}{V_A}\binom{N-N_A}{V_B} F(\hat{c}_A)^{V_A}\left(1-F(\hat{c}_A)\right)^{N_A-V_A} F(\hat{c}_B)^{V_B}\left(1-F(\hat{c}_B)\right)^{N-N_A-V_B}(\pi_A(V_A,V_B,N_A)),$$

where

$$\pi_A(V_A,V_B,N_A) = \begin{cases} [1+\gamma_A^A N_A + \gamma_A^B(N-N_A)]\left[G\left(\frac{V_A+1}{V_A+V_B+1}\right) - G\left(\frac{V_A}{V_A+V_B}\right)\right] & if\ V_A+V_B>0 \\ \frac{1}{2}[1+\gamma_A^A N_A + \gamma_A^B(N-N_A)] & if\ V_A+V_B=0 \end{cases}.$$

Similarly $E[B_B(\hat{c}_A,\hat{c}_B)]$ represents the expected gross benefit from voting for a supporter of party B and is calculated in an analogous way. For this to be an equilibrium, it must be the case that the supporter of A with cost $\hat{c}_A$ and the supporter of B with cost $\hat{c}_B$ must be indifferent between voting and abstaining when all other players adopt the same strategy.

To prove the existence of such equilibrium we construct the function

$$\Phi(\hat{c}_A, \hat{c}_B) = \left(\max\left\{\min\{ E[B_A(\hat{c}_A, \hat{c}_B)], \overline{c}_A\}, \underline{c}_A\right\}, \max\left\{\min\{E[B_B(\hat{c}_A, \hat{c}_B)], \overline{c}_B\}, \underline{c}_B\right\}\right).$$

As both $E[B_A(\hat{c}_A, \hat{c}_B)]$ and $E[B_B(\hat{c}_A, \hat{c}_B)]$ are continuous in $\hat{c}_A$ and $\hat{c}_B$, then $\Phi(\hat{c}_A, \hat{c}_B)$ is also continuous. Hence, by Brower's fixed point theorem there must exist a pair (c_A^*, c_B^*) such that $\Phi(c_A^*, c_B^*) = (c_A^*, c_B^*)$. If $c_P^* \in \left(\underline{c}_P, \overline{c}_P\right)$, then all supporters of party P with cost less than c_P^* will vote and those with higher costs will abstain. Similarly, if $c_P^* = \underline{c}_P$ all members of party P will abstain, while if $c_P^* = \overline{c}_P$ they will all vote.■

Proof of Proposition 2

Notice that the expectation in Equation (2) is taken over the random variables $\hat{\lambda}$, $\hat{\nu}_A$ and $\hat{\nu}_B$. Next, note that the limiting distributions of $\hat{\lambda}$, $\hat{\nu}_A\hat{\lambda}$ and $\hat{\nu}_B\left(1-\hat{\lambda}\right)$ are $N\left[\lambda, \frac{\lambda(1-\lambda)}{N}\right]$, $N\left[\nu_A\lambda, \frac{\nu_A\lambda(1-\nu_A\lambda)}{N}\right]$ and $N\left[\nu_B(1-\lambda), \frac{\nu_B(1-\lambda)[1-\nu_B(1-\lambda)]}{N}\right]$, respectively. As a result we have $Plim\ \hat{\lambda} = \lambda$, $Plim\ \hat{\nu}_A\hat{\lambda} = \nu_A\lambda$ and $Plim\ \hat{\nu}_B\left(1-\hat{\lambda}\right) = \nu_B(1-\lambda)$. Hence, supposing an equilibrium characterized by thresholds c_A^* and c_B^*, turnout in equilibrium is equal $\lambda F_A(c_A^*) + (1-\lambda)F_B(c_B^*)$. In order to evaluate the limit of Equation (2), we must consider *two cases*.

i) In the *first case*, suppose that $c_A^* = \underline{c}_A$ and $c_B^* = \underline{c}_B$, implying that $\nu_A = \nu_B = 0$. Here, all other citizens choose *not* to vote with probability 1. As a result, the returns to voting for a supporter of party A are given by:

$$E[B_A|\nu_A + \nu_B = 0] = E\left[\tfrac{1}{2}\left(1 + N\hat{\Gamma}_A\right)\right].$$

Given that $\hat{\lambda} \to \lambda$, in the limit, as $N \to \infty$, then $\hat{\Gamma}_A$ converges to Γ_A. Hence the returns to voting become infinite if $\Gamma_A > 0$, while they are equal to $\frac{1}{2}$ when $\Gamma_A = 0$. Clearly, the same holds for a supporter of party B, with the gross benefit from voting equal to either infinity or $\frac{1}{2}$ when $\Gamma_B > 0$

or $\Gamma_B = 0$, respectively. If $\Gamma_A = \Gamma_B = 0$, this implies that $c_A^* = \underline{c}_A$ and $c_B^* = \underline{c}_B$ can only be an equilibrium if $\underline{c}_A \geq \frac{1}{2}$ and $\underline{c}_B \geq \frac{1}{2}$. On the other hand, if at least one between Γ_A and Γ_B is strictly positive, then it cannot be the case that $c_A^* = \underline{c}_A$ and $c_B^* = \underline{c}_B$ in equilibrium.

ii) In the *second case*, suppose $c_P^* > \underline{c}_P$ for at least for one party. As a result, for at least one type the probability of voting is strictly positive, which translates into $\nu_A + \nu_B > 0$. We begin by showing that for this case the Plim of the second term in Equation (2) is zero and can thus be ignored. To see this, first note that the second half of this term is the probability that no one votes. Following the approach taken by Taylor and Yildirim (2010) it is easy to show that the limiting marginal distributions of $\{\hat{\nu}_A\hat{\lambda}N, \hat{\nu}_B(1-\hat{\lambda})N\}$ are independent Poisson distributions with means equal to $\{\nu_A\lambda N, \nu_B(1-\lambda)N\}$.[6] As a result,

$$\lim_{N\to\infty} P\left[\hat{\nu}_A\hat{\lambda}N + \hat{\nu}_B(1-\hat{\lambda})N = 0\right] = \frac{(\nu_A\lambda N)^0}{0!e^{\nu_A\lambda N}}\frac{[\nu_B(1-\lambda)N]^0}{0!e^{\nu_B(1-\lambda)N}} = \frac{1}{e^{N[\nu_A\lambda+\nu_B(1-\lambda)]}}.$$

Next assume, without loss of generality, that $\gamma_A^A \geq \gamma_A^B$ and note that $\lambda < 1$. Thus,

$$\lim_{N\to\infty} E\left[\frac{1}{2}\left(1+N\hat{\Gamma}_A\right)\middle|\hat{\nu}_A\hat{\lambda}N + \hat{\nu}_B\left(1-\hat{\lambda}\right)N = 0\right] * P\left[\hat{\nu}_A\hat{\lambda}N + \hat{\nu}_B\left(1-\hat{\lambda}\right)N = 0\right] \leq$$

$$\lim N\rightarrow\infty 1+\gamma AAN2eN\nu A\lambda+\nu B(1-\lambda)=0.$$

We now turn to the first term in Equation (2). For simplicity of notation let us define

$$\Delta = \frac{\hat{\nu}_B(1-\hat{\lambda})}{[\hat{\nu}_A\hat{\lambda}N+\hat{\nu}_B(1-\hat{\lambda})N+1][\hat{\nu}_A\hat{\lambda}+\hat{\nu}_B(1-\hat{\lambda})]}.$$

We can re-write the expression over which we are taking the expectation as:

$$\left(\frac{1}{N}+\hat{\Gamma}_A\right)N\Delta\frac{\left[G\left(\frac{\hat{\nu}_A\hat{\lambda}}{\hat{\nu}_A\hat{\lambda}+\hat{\nu}_B(1-\hat{\lambda})}+\Delta\right)-G\left(\frac{\hat{\nu}_A\hat{\lambda}}{\hat{\nu}_A\hat{\lambda}+\hat{\nu}_B(1-\hat{\lambda})}\right)\right]}{\Delta}.$$

Recall that, by construction, this term is limited to outcomes where $\hat{\nu}_A\hat{\lambda}N + \hat{\nu}_B\left(1-\hat{\lambda}\right)N > 0$.

Moreover, notice that, by definition:

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[6] See Taylor & Yildirim (2010), Lemma 5.

$$\lim_{N\to\infty} \frac{\left[G\left(\frac{\hat{v}_A\hat{\lambda}N}{\hat{v}_A\hat{\lambda}N+\hat{v}_B(1-\hat{\lambda})N}+\Delta\right)-G\left(\frac{\hat{v}_A\hat{\lambda}N}{\hat{v}_A\hat{\lambda}N+\hat{v}_B(1-\hat{\lambda})N}\right)\right]}{\Delta} = G'\left(\frac{\hat{v}_A\hat{\lambda}N}{\hat{v}_A\hat{\lambda}N+\hat{v}_B(1-\hat{\lambda})N}\right).$$

Thus we have the result of Equation (A1).

$$\lim_{N\to\infty}\left(\frac{1}{N}+\hat{\Gamma}_A\right)N\Delta\frac{\left[G\left(\frac{\hat{v}_A\hat{\lambda}}{\hat{v}_A\hat{\lambda}+\hat{v}_B(1-\hat{\lambda})}+\Delta\right)-G\left(\frac{\hat{v}_A\hat{\lambda}}{\hat{v}_A\hat{\lambda}+\hat{v}_B(1-\hat{\lambda})}\right)\right]}{\Delta}\Bigg|\,\hat{v}_A\hat{\lambda}N+\hat{v}_B\left(1-\hat{\lambda}\right)N>0 =$$

$$\hat{\Gamma}_A\frac{\hat{v}_B(1-\hat{\lambda})}{\left[\hat{v}_A\hat{\lambda}+\hat{v}_B(1-\hat{\lambda})\right]^2}G'\left(\frac{\hat{v}_A\hat{\lambda}}{\hat{v}_A\hat{\lambda}+\hat{v}_B(1-\hat{\lambda})}\right). \tag{A1}$$

Recall that the $Plim\ \hat{\lambda}=\lambda$, $Plim\ \hat{v}_A\hat{\lambda}=\nu_A\lambda$ and $Plim\ \hat{v}_B\left(1-\hat{\lambda}\right)=\nu_B(1-\lambda)$. This fact, combined with the result that the second term of Equation (2) converges to zero, implies that, conditional on $\hat{v}_A\hat{\lambda}N+\hat{v}_B\left(1-\hat{\lambda}\right)N>0$, in the limit Equation (2) collapses to

$$\Gamma_A\frac{\nu_B(1-\lambda)}{[\nu_A\lambda+\nu_B(1-\lambda)]^2}G'\left(\frac{\nu_A\lambda}{\nu_A\lambda+\nu_B(1-\lambda)}\right)$$

Finally, recalling that in equilibrium individuals use a threshold voting strategy, the probability ν_P that a member of the generic party P votes is equal to $F_P(c_P^*)$. Hence, we can re-write the limiting benefit for A-members as

$$\Gamma_A\frac{F_B(c_B^*)(1-\lambda)}{\left[F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)\right]^2}G'\left(\frac{F_A(c_A^*)\lambda}{F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)}\right). \tag{A2}$$

The limiting benefit for B-members can be calculated in an analogous way and is equal to

$$\Gamma_B\frac{F_A(c_A^*)\lambda}{\left[F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)\right]^2}G'\left(\frac{F_B(c_B^*)(1-\lambda)}{F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)}\right). \tag{A3}$$

This implies that, in equilibrium, we have

$$c_A^*=\Gamma_A\frac{F_B(c_B^*)(1-\lambda)}{\left[F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)\right]^2}G'\left(\frac{F_A(c_A^*)\lambda}{F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)}\right), \tag{A4}$$

and

$$c_B^*=\Gamma_B\frac{F_A(c_A^*)\lambda}{\left[F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)\right]^2}G'\left(\frac{F_B(c_B^*)(1-\lambda)}{F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)}\right). \tag{A5}$$

Suppose $\Gamma_A,\Gamma_B>0$. We are going to show that equilibrium turnout is bounded away from zero.

Notice first that, by definition,

$$G'\left(\frac{F_A(c_A^*)\lambda}{F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)}\right) = G'\left(\frac{F_B(c_B^*)(1-\lambda)}{F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)}\right).$$

Therefore, from equations (A4) and (A5) we know that, in equilibrium, the following holds

$$\frac{c_A^* F_A(c_A^*)}{c_B^* F_B(c_B^*)} = \frac{\Gamma_A(1-\lambda)}{\Gamma_B\lambda} = \frac{\gamma_A^A\lambda\,(1-\lambda)+\gamma_A^B(1-\lambda)^2}{\gamma_B^A\,\lambda^2+\gamma_B^B\lambda(1-\lambda)}. \tag{A6}$$

From (A6) it follows that $\frac{\partial c_A^*}{\partial c_B^*} > 0$ and, as a consequence, there exists a function ϕ such that

$c_B^* = \phi(c_A^*)$ and $\frac{\partial\phi(c_A^*)}{\partial c_A^*} > 0$. This means that $F_B\big(\phi(c_A^*)\big) \to 0$ when $c_A^* \to \underline{c}_A$.

Suppose that $c_A^* \to \underline{c}_A$ when $N \to \infty$. This implies that, at the limit, the benefit from voting for an A-member is less than or equal to $\underline{c}_A$. Notice from (A2) that, since $G'(.)$ is finite and strictly positive, this is only possible if

$$\lim_{c_A^*\to\underline{c}_A} \frac{F_B(c_B^*)(1-\lambda)}{\left[F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)\right]^2} = \frac{F_B\big(\phi(c_A^*)\big)}{\big(F_A(c_A^*)\big)^2}\frac{(1-\lambda)}{\lambda^2} = k < \infty. \tag{A7}$$

On the other hand we know from (A3) that, when $c_A^* \to \underline{c}_A$, the benefit for a B-member tends to

$$\frac{F_A(c_A^*)}{\Big(F_B\big(\phi(c_A^*)\big)\Big)^2}\frac{\lambda}{(1-\lambda)^2}\Gamma_B G'(.). \tag{A8}$$

Note that, if the condition outlined in (A7) holds, it must be the case that $F_B\big(\phi(c_A^*)\big)$ goes to zero faster than $F_A(c_A^*)$. However, this implies that expression (A8) tends to infinity and thus each B-supporter votes in equilibrium. Analogously, it can be shown that if $c_B^* \to \underline{c}_B$ then the benefit from voting for an A-member tends to infinity, proving that turnout is bounded away from zero in equilibrium.

Finally, suppose $\Gamma_P = 0$. In this case, the limiting benefit from voting for a member of party P is equal to zero, which means that c_P^* cannot be greater than $\underline{c}_P$ when $N \to \infty$. This in turns implies that the benefit from voting for a member of party $\overline{P}$ tends to zero, even if $\Gamma_{\overline{P}} > 0$, and as a consequence $c_{\overline{P}}^{*} \rightarrow \underline{c}_{\overline{P}}$. Therefore, unless both Γ_A and Γ_B are strictly positive, equilibrium turnout is either zero, if $\Gamma_A = \Gamma_B = 0$, $\underline{c}_A \geq \frac{1}{2}$ and $\underline{c}_B \geq \frac{1}{2}$, or converges to zero otherwise. ■

Proof of Lemma 1

The fact that $\frac{\partial(c_A^* F_A(c_A^*)/c_B^* F_B(c_B^*))}{\partial \gamma_A^P} > 0$ and $\frac{\partial(c_A^* F_A(c_A^*)/c_B^* F_B(c_B^*))}{\partial \gamma_B^P} < 0$ follows immediately from (A6). Moreover, notice that $\frac{\partial(c_A^* F_A(c_A^*)/c_B^* F_B(c_B^*))}{\partial \gamma_A^P}$ and $\frac{\partial(c_A^* F_A(c_A^*)/c_B^* F_B(c_B^*))}{\partial \gamma_B^P}$ can be re-written, respectively, as

$$\frac{1}{\left(c_B^* F_B(c_B^*)\right)^2}\left[\frac{\partial c_A^*}{\partial \gamma_A^P}\left(\frac{1}{c_A^*} + \frac{\partial F_A(c_A^*)}{\partial c_A^*}\frac{1}{F_A(c_A^*)}\right) - \frac{\partial c_B^*}{\partial \gamma_A^P}\left(\frac{1}{c_B^*} + \frac{\partial F_B(c_B^*)}{\partial c_B^*}\frac{1}{F_B(c_B^*)}\right)\right] \tag{A9}$$

and

$$\frac{1}{\left(c_B^* F_B(c_B^*)\right)^2}\left[\frac{\partial c_A^*}{\partial \gamma_B^P}\left(\frac{1}{c_A^*} + \frac{\partial F_A(c_A^*)}{\partial c_A^*}\frac{1}{F_A(c_A^*)}\right) - \frac{\partial c_B^*}{\partial \gamma_B^P}\left(\frac{1}{c_B^*} + \frac{\partial F_B(c_B^*)}{\partial c_B^*}\frac{1}{F_B(c_B^*)}\right)\right]. \tag{A10}$$

Therefore, we know from (A9) that

$$\frac{\partial c_A^*}{\partial \gamma_A^P}\left(\frac{1}{c_A^*} + \frac{\partial F_A(c_A^*)}{\partial c_A^*}\frac{1}{F_A(c_A^*)}\right) > \frac{\partial c_B^*}{\partial \gamma_A^P}\left(\frac{1}{c_B^*} + \frac{\partial F_B(c_B^*)}{\partial c_B^*}\frac{1}{F_B(c_B^*)}\right).$$

Suppose $\frac{\partial c_A^*}{\partial \gamma_A^P} < 0$ and $\frac{\partial c_B^*}{\partial \gamma_A^P} > 0$. The left hand side of the above inequality would be negative, while the right hand side would be positive, which is a contradiction.

Similarly, we know from (A10) that

$$\frac{\partial c_A^*}{\partial \gamma_B^P}\left(\frac{1}{c_A^*} + \frac{\partial F_A(c_A^*)}{\partial c_A^*}\frac{1}{F_A(c_A^*)}\right) < \frac{\partial c_B^*}{\partial \gamma_B^P}\left(\frac{1}{c_B^*} + \frac{\partial F_B(c_B^*)}{\partial c_B^*}\frac{1}{F_B(c_B^*)}\right).$$

Suppose $\frac{\partial c_A^*}{\partial \gamma_B^P} > 0$ and $\frac{\partial c_B^*}{\partial \gamma_B^P} < 0$. The left hand side would be positive and the right hand side would be negative, which cannot be. ■

Proof of Lemma 2

From (A6) we calculate $\frac{\partial(c_A^* F_A(c_A^*)/c_B^* F_B(c_B^*))}{\partial \lambda}$ which is equal to

$$\frac{\gamma_A^A(1-2\lambda)-2\gamma_A^B(1-\lambda)}{\lambda[\gamma_B^A\lambda+\gamma_B^B(1-\lambda)]}-\frac{(1-\lambda)[\gamma_A^A\lambda+\gamma_A^B(1-\lambda)][2\gamma_B^A\lambda+\gamma_B^B(1-2\lambda)]}{\lambda^2[\gamma_B^A\lambda+\gamma_B^B(1-\lambda)]^2}.$$

The above expression reduces to

$$\frac{-\gamma_A^B\gamma_B^B(1-\lambda)^2-2\gamma_A^B\gamma_B^A\lambda(1-\lambda)-\gamma_A^A\gamma_B^A\lambda^2}{\lambda^2[\gamma_B^A\lambda+\gamma_B^B(1-\lambda)]^2}. \qquad \text{(A11)}$$

From (A11) we can see that the sign of the derivative is zero if $\gamma_A^B=\gamma_B^A=0$ (i.e. citizens are Group Minded), but it is negative otherwise.

Notice that $\frac{\partial(c_A^*F_A(c_A^*)/c_B^*F_B(c_B^*))}{\partial\lambda}$ equals

$$\frac{1}{\left(c_B^*F_B(c_B^*)\right)^2}\left[\frac{\partial c_A^*}{\partial\lambda}\left(\frac{1}{c_A^*}+\frac{\partial F_A(c_A^*)}{\partial c_A^*}\frac{1}{F_A(c_A^*)}\right)-\frac{\partial c_B^*}{\partial\lambda}\left(\frac{1}{c_B^*}+\frac{\partial F_B(c_B^*)}{\partial c_B^*}\frac{1}{F_B(c_B^*)}\right)\right].$$

Thus, if voters are Paternalistic we know that

$$\frac{\partial c_A^*}{\partial\lambda}\left(\frac{1}{c_A^*}+\frac{\partial F_A(c_A^*)}{\partial c_A^*}\frac{1}{F_A(c_A^*)}\right)<\frac{\partial c_B^*}{\partial\lambda}\left(\frac{1}{c_B^*}+\frac{\partial F_B(c_B^*)}{\partial c_B^*}\frac{1}{F_B(c_B^*)}\right).$$

Suppose $\frac{\partial c_A^*}{\partial\lambda}>0$ and $\frac{\partial c_B^*}{\partial\lambda}<0$. The left hand side of the above inequality would be positive, while the right hand side would be negative, which cannot be.

Finally, if citizens are Group Minded we have

$$\frac{\partial c_A^*}{\partial\lambda}\left(\frac{1}{c_A^*}+\frac{\partial F_A(c_A^*)}{\partial c_A^*}\frac{1}{F_A(c_A^*)}\right)=\frac{\partial c_B^*}{\partial\lambda}\left(\frac{1}{c_B^*}+\frac{\partial F_B(c_B^*)}{\partial c_B^*}\frac{1}{F_B(c_B^*)}\right).$$

Notice that $\frac{1}{c_A^*}+\frac{\partial F_A(c_A^*)}{\partial c_A^*}\frac{1}{F_A(c_A^*)}>0$ and $\frac{1}{c_B^*}+\frac{\partial F_B(c_B^*)}{\partial c_B^*}\frac{1}{F_B(c_B^*)}>0$. Therefore it must be $\frac{\partial c_A^*}{\partial\lambda}=\frac{\partial c_B^*}{\partial\lambda}$. ■

Proof of Proposition 3

i. We know from Proposition 2 that

$$c_B^*[F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)]^2=\Gamma_B F_A(c_A^*)\lambda G'\left(\frac{F_B(c_B^*)(1-\lambda)}{F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)}\right), \qquad \text{(A12)}$$

and

$$c_A^*[F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)]^2=\Gamma_A F_B(c_B^*)(1-\lambda)G'\left(\frac{F_A(c_A^*)\lambda}{F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)}\right). \qquad \text{(A13)}$$

When $\lambda \to 0$, Equation (A12) reduces to $c_B^* F_B(c_B^*)^2 = 0$ and therefore

$$\lim_{\lambda \to 0} F_B(c_B^*) = 0. \tag{A14}$$

As a consequence, if voters are Paternalistic, the benefit for an A-member converges to $\gamma_A^B \frac{1}{F_B(c_B^*)} G'\left(\frac{0}{F_B(c_B^*)}\right)$ when $\lambda \to 0$. Since $G'(.)$ is always positive and finite, the latter tends to infinity and $\lim_{\lambda \to 0} F_A(c_A^*) = 1$. Analogously Equation (A13) reduces to $c_A^* F_A(c_A^*)^2 = 0$ when $\lambda \to 1$, implying that

$$\lim_{\lambda \to 1} F_A(c_A^*) = 0. \tag{A15}$$

Therefore, if voters are Paternalistic, when $\lambda \to 1$ the benefit for a B-member converges to $\gamma_B^A \frac{1}{F_A(c_A^*)} G'\left(\frac{0}{F_A(c_A^*)}\right)$ and $\lim_{\lambda \to 1} F_B(c_B^*) = 1$.

ii. When voters are Group Minded, we know from (A6) that $\frac{c_A^* F_A(c_A^*)}{c_B^* F_B(c_B^*)} = \frac{\gamma_A^A}{\gamma_B^B}$. This, together with (A14), implies that $\lim_{\lambda \to 0} F_B(c_B^*) = \lim_{\lambda \to 0} F_A(c_A^*) = 0$. Similarly, given (A15), we conclude that $\lim_{\lambda \to 1} F_A(c_A^*) = \lim_{\lambda \to 1} F_B(c_B^*) = 0$. ■

Proof of Proposition 4

Given Proposition 3, we know that, by continuity, there must exist at least one value $\tilde{\lambda}$ such that $c_A^*(\tilde{\lambda}) = c_B^*(\tilde{\lambda})$. Moreover, we know from Lemma 2 that $\frac{\partial\left(\frac{c_A^* F(c_A^*)}{c_B^* F(c_B^*)}\right)}{\partial \lambda} < 0$ and, therefore, when $c_A^* = c_B^*$ it must be the case that $\frac{\partial c_B^*}{\partial \lambda} > \frac{\partial c_A^*}{\partial \lambda}$. However, if there were more than one crossing point, it would imply $\frac{\partial c_B^*}{\partial \lambda} < \frac{\partial c_A^*}{\partial \lambda}$ for at least one of these points, which cannot be. This proves that there exists a unique $\tilde{\lambda}$ such that $c_A^*(\tilde{\lambda}) = c_B^*(\tilde{\lambda})$, $c_A^*(\lambda) > c_B^*(\lambda) \forall \lambda < \tilde{\lambda}$ and $c_A^*(\lambda) < c_B^*(\lambda) \forall \lambda > \tilde{\lambda}$.

Suppose $\lambda = \frac{1}{2}$. From (A6) we know that $\frac{c_A^* F(c_A^*)}{c_B^* F(c_B^*)} = \frac{\gamma_A^A + \gamma_A^B}{\gamma_B^A + \gamma_B^B}$. Hence, if $\gamma_A^A + \gamma_A^B = \gamma_B^A + \gamma_B^B$ then $c_A^*\left(\lambda = \frac{1}{2}\right) = c_B^*\left(\lambda = \frac{1}{2}\right)$, implying that $\tilde{\lambda} = \frac{1}{2}$. If $\gamma_A^A + \gamma_A^B < \gamma_B^A + \gamma_B^B$ then $c_A^*\left(\lambda = \frac{1}{2}\right) <$

$c_B^*\left(\lambda = \frac{1}{2}\right)$, which implies that $\tilde{\lambda} < \frac{1}{2}$. Finally, if $\gamma_A^A + \gamma_A^B > \gamma_B^A + \gamma_B^B$ then $c_A^*\left(\lambda = \frac{1}{2}\right) >$ $c_B^*\left(\lambda = \frac{1}{2}\right)$, meaning that $\tilde{\lambda} > \frac{1}{2}$. ■

Proof of Proposition 5

Recall from Proposition 3 that, when voters are Group Minded, $\frac{c_A^* F(c_A^*)}{c_B^* F(c_B^*)} = \frac{\gamma_A^A}{\gamma_B^B}$. It follows immediately that if $\gamma_A^A > \gamma_B^B$ then $c_A^*(\lambda) > c_B^*(\lambda) \forall \lambda$, while $c_A^*(\lambda) < c_B^*(\lambda) \forall \lambda$ if $\gamma_A^A < \gamma_B^B$ and $c_A^*(\lambda) = c_B^*(\lambda) \forall \lambda$ if $\gamma_A^A = \gamma_B^B$. ■

Proof of Proposition 6

Consider first the case of Group Minded voters. Since $\gamma_A^B = \gamma_B^A = 0$, assuming $\gamma_A^A + \gamma_A^B = \gamma_B^A + \gamma_B^B$ implies $\gamma_A^A = \gamma_B^B$. We know from Proposition 5 that in this case $c_A^*(\lambda) = c_B^*(\lambda) \forall \lambda$. It follows that $\theta_A^*(\lambda) < \frac{1}{2}\ \forall \lambda < \frac{1}{2}$ and $\theta_A^*(\lambda) > \frac{1}{2}\ \forall \lambda > \frac{1}{2}$.

Consider now the case of Paternalistic voters. From (A6) we know that

$$\lambda c_A^* F(c_A^*)[\lambda \gamma_B^A + (1-\lambda)\gamma_B^B] = (1-\lambda) c_B^* F(c_B^*)[\lambda \gamma_A^A + (1-\lambda)\gamma_A^B]. \tag{A16}$$

Let us re-arrange $\gamma_A^A + \gamma_A^B = \gamma_B^A + \gamma_B^B$ as $\gamma_A^A - \gamma_B^A = \gamma_B^B - \gamma_A^B$. As we are assuming that $\gamma_A^A > \gamma_B^A$ and $\gamma_B^B > \gamma_A^B$, this implies that $\lambda(\gamma_A^A - \gamma_B^A) < (1-\lambda)(\gamma_B^B - \gamma_A^B)$ when $\lambda < \frac{1}{2}$, while the opposite is true when $\lambda > \frac{1}{2}$. Re-arranging we have $\lambda \gamma_B^A + (1-\lambda)\gamma_B^B > \lambda \gamma_A^A + (1-\lambda)\gamma_A^B$ if $\lambda < \frac{1}{2}$ and $\lambda \gamma_B^A + (1-\lambda)\gamma_B^B < \lambda \gamma_A^A + (1-\lambda)\gamma_A^B$ for $\lambda > \frac{1}{2}$. In addition, we know that $c_A^* > c_B^*$ for $\lambda < \frac{1}{2}$ and vice versa. Hence, from (A16) we can conclude that $\lambda F(c_A^*) < (1-\lambda)F(c_B^*)$ when $\lambda < \frac{1}{2}$ and $\lambda F(c_A^*) > (1-\lambda)F(c_B^*)$ when $\lambda > \frac{1}{2}$. As a consequence $\theta_A^*(\lambda) < \frac{1}{2}\ \forall \lambda < \frac{1}{2}$ and $\theta_A^*(\lambda) > \frac{1}{2}\ \forall \lambda > \frac{1}{2}$.■

Proof of Proposition 7

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Proof of Proposition 8

Consider the benefit from voting of an A-member and differentiate it with respect to c_A^*. We obtain

$$(1-\lambda)\Gamma_A\left\{X - G'\left(\frac{F(c_A^*)\lambda}{F(c_A^*)\lambda+F(c_B^*)(1-\lambda)}\right)Y\right\},$$

where $X = G''\left(\frac{F(c_A^*)\lambda}{F(c_A^*)\lambda+F(c_B^*)(1-\lambda)}\right)\lambda(1-\lambda)F(c_B^*)\frac{\frac{\partial F(c_A^*)}{\partial c_A^*}F(c_B^*)-\frac{\partial F(c_B^*)}{\partial c_A^*}F(c_A^*)}{\left[F(c_A^*)\lambda+F(c_B^*)(1-\lambda)\right]^4}$ and

$$Y = \frac{\frac{\partial F(c_B^*)}{\partial c_A^*}[\lambda F(c_A^*)-(1-\lambda)F(c_B^*)]-2\lambda\frac{\partial F(c_A^*)}{\partial c_A^*}F(c_B^*)}{\left[F(c_A^*)\lambda+F(c_B^*)(1-\lambda)\right]^3}.$$

First of all, recall from the proof of Proposition 2 that $\frac{\partial c_A^*}{\partial c_B^*} > 0$ and, as consequence, $\frac{\partial F(c_B^*)}{\partial c_A^*} > 0$.

Let us first focus on $\lambda < \frac{1}{2}$. Since $\gamma_A^A + \gamma_A^B = \gamma_B^A + \gamma_B^B$, $\gamma_A^A > \gamma_B^A$ and $\gamma_B^B > \gamma_A^B$ we know from Proposition 6 that $\lambda F(c_A^*) < (1-\lambda)F(c_B^*)$. It follows that $G''(.) > 0$ and, therefore, $X < 0$ if $\frac{\partial F(c_A^*)}{\partial c_A^*}F(c_B^*) < \frac{\partial F(c_B^*)}{\partial c_A^*}F(c_A^*)$. Moreover, $\lambda F(c_A^*) < (1-\lambda)F(c_B^*)$ implies that $Y < 0$.

Consider now the case $\lambda > \frac{1}{2}$. From Proposition 6 we know that $\lambda F(c_A^*) > (1-\lambda)F(c_B^*)$, implying that $G''(.) < 0$. Hence, $\frac{\partial F(c_A^*)}{\partial c_A^*}F(c_B^*) > \frac{\partial F(c_B^*)}{\partial c_A^*}F(c_A^*)$ is a sufficient condition for $X < 0$.

In addition, let us re-write Y as

$$\frac{\lambda\left(\frac{\partial F(c_B^*)}{\partial c_A^*}F(c_A^*)-2\frac{\partial F(c_A^*)}{\partial c_A^*}F(c_B^*)\right)-\frac{\partial F(c_B^*)}{\partial c_A^*}(1-\lambda)F(c_B^*)}{\left[F(c_A^*)\lambda+F(c_B^*)(1-\lambda)\right]^3}.$$

Notice that $Y < 0$ if $\frac{\partial F(c_A^*)}{\partial c_A^*}F(c_B^*) > \frac{\partial F(c_B^*)}{\partial c_A^*}F(c_A^*)$.

Recall from Proposition 4 that $\gamma_A^A + \gamma_A^B = \gamma_B^A + \gamma_B^B$ implies $c_A^* > c_B^* \forall\lambda < \frac{1}{2}$ and $c_A^* < c_B^* \forall\lambda > \frac{1}{2}$.

This means that $\frac{\frac{\partial F(c_A^*)}{\partial c_A^*}}{F(c_A^*)} < \frac{\frac{\partial F(c_B^*)}{\partial c_A^*}}{F(c_B^*)} \forall c_A^* > c_B^*$ and $\frac{\frac{\partial F(c_A^*)}{\partial c_A^*}}{F(c_A^*)} > \frac{\frac{\partial F(c_B^*)}{\partial c_A^*}}{F(c_B^*)} \forall c_A^* < c_B^*$ are sufficient conditions for the benefit of an A-member to be weakly decreasing in c_A^* on the intervals $\lambda \in (0, \frac{1}{2})$ and $\lambda \in (\frac{1}{2}, 1)$. These conditions translate into $\partial \frac{\frac{\partial F(c)}{\partial c}/F(c)}{\partial c} < 0$. The latter can be expressed as $\frac{F''F - F'F'}{F^2} < 0$ and thus $\frac{F''}{F'} < \frac{F'}{F}$.

Finally, note that $c_A^* = c_B^*$ when $\lambda = \frac{1}{2}$ and therefore $\frac{\partial F(c_A^*)}{\partial c_A^*} F(c_B^*) = \frac{\partial F(c_B^*)}{\partial c_A^*} F(c_A^*)$ and $\lambda F(c_A^*) = (1-\lambda) F(c_B^*)$, implying $X = 0$ and $Y = 0$. Thus, in this case, the derivative of the benefit is equal to zero. If the benefit for an A-member is non-increasing in c_A^* it means that there exists at most one fixed point. Having already proved the existence of an equilibrium, this demonstrates that, given the assumptions, $\frac{F''}{F'} < \frac{F'}{F}$ is a sufficient condition for its uniqueness. ■

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