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Self-insurance and the samaritan’s dilemma in a federation

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Self-Insurance and the Samaritan's Dilemma in a Federation

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Abstract

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Keywords: Fiscal Federalism, Self-Insurance, Public Goods
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1 Introduction

There are many reasons why smaller communities, villages, cities, regions decide to give up some independence and to join a bigger community. Two of the most pervasive arguments for joining a federation are the more efficient provision of public goods, which are non-rival at the federal level, and the sharing of risk among risk averse regions. In general, both arguments call for more than one level of government. In the public good case, having several jurisdictions leads to a trade-off between the efficient level of public good provision and the heterogeneity of the population. When sharing risk, having a superior jurisdiction allows regions to commit themselves to honor the risk sharing arrangement, even if ex-post the region not affected by the loss has an incentive to defect.[1]

The present paper focuses on the risk sharing aspect in a federation. Consider regions or states within a country or a federation that can be hit by a shock leading to losses that occur with an exogenous probability and in a stochastically independent way.[2] For instance, the Japanese regions of Tokyo and Osaka are at high risk of an earthquake or the US states of Missouri and Louisiana face the risk of floods. In both examples, the regions are part of a superior jurisdiction which allows risk consolidation on an aggregate level. This has a positive effect on all regions' expected payoff, if the regions are risk averse. Apart from that, an individual region can also undertake investments to lower the size of the loss. In the case of an earthquake, the region can impose appropriately stable building regulations. For flood protection, the region can build levees and dikes to prevent flooding. In both cases, the investments to reduce the size of the loss are costly and reduce income. Such a situation where the probability of the loss is given but the size of the loss can be influenced by investing in a public good is a self-insurance situation (Ehrlich and Becker, 1972). Being part of a federation, there is a double incentive not to invest in self-insurance. First, in case disaster strikes, the affected region can rely on support from the central government, which redistributes income from the region not being hit by a shock to the region hit by a shock. Second, due to this insurance effect within the federation, the investment in self-insurance becomes a public good and the regions contribute to this public good privately. Under normal circumstances, there is underprovision of this public good. A similar situation is faced by sovereign countries and supra-national organizations. If all countries face a potential loss, the countries are better off if they pool the risk. However, the independent countries face the same incentive problem as the dependent regions. Individually, each country has an incentive to minimize the own investment in loss reduction, while from a common point of view it would still be efficient to invest in the public insurance good. The problem is exacerbated here because the individual sovereign countries cannot in principle be coerced to contribute to the public good, so participation in the insurance scheme must be ex ante voluntary. One can think of international agreements as delivering a self-commitment device that ex-ante makes all individual countries better off.

In this paper we study the interaction between public self-insurance and central government's transfers. We compare the first-best situation where the central government can also determine the

[1] See Kocherlakota (1996) and Genicot and Ray (2003) for an analysis of self-enforcing risk sharing arrangements without commitment.

[2] Throughout the paper, we will focus without loss of generality on regions within a federation, but our analysis also applies to boroughs joining a city or to sovereign countries joining a supra-national organization like the European Union.

regional government's choices with a second-best situation where the regional government is free to act under the transfer system the central government has chosen. We have no explicit public goods and no related direct spillover effect. Our simple model has no taxation and no redistribution and purely selfish regions. Uncertainty can be mitigated by self-insurance investment, and it is this self-insurance effort which has a public good character. Given the insurance effect in a federation due to risk consolidation, the intuitive result follows that in a federation the first-best self-insurance level is always smaller than in an autarky situation. Since the cost of the loss is shared, risk averse regions are willing to take bigger risks, i. e., they reduce their self-insurance efforts. However, given the federal transfer mechanism, the individual regions have an incentive to further reduce the self-insurance effort suboptimally if there is no central government with full information and coercive power. The Nash equilibrium of the private contributions to public self-insurance in a non-cooperative setting is always smaller than the first-best equilibrium level. If the central government takes this incentive situation into account, it will redesign the regional insurance mechanism with the aim of increasing the self-insurance effort choice of the individual regions. The resulting second-best self-insurance efforts in the non-cooperative setting will still be lower than the first-best effort level, leading to second-best redistribution transfers that are always larger (in absolute terms) than the corresponding first-best transfers. The welfare results are ambiguous and being member of a federation is not always in the interest of the individual region. On the positive side, there is the consolidation of risk. On the negative side, this risk reduction creates a Samaritan's Dilemma (Buchanan, 1975) between insurance and redistribution by reducing the incentive to invest in public self-insurance.[3] Which effect prevails depends on the regional payoff functions and on the probabilities of the loss. It follows that some risks are better pooled, some are not. A possible solution is to fix the redistribution transfers independently of the loss. For fixed transfers, the central government may reach the first-best outcome.

The issue of risk sharing in a federation has been studied both empirically and theoretically. Sala-I-Martin and Sachs (1992) and Asdrubali et al. (1996) analyze risk sharing among US states. They find that the capital market and the credit market appear to be the most important mechanism for risk sharing. However, both papers consider the federal government as an important complement for risk sharing. Persson and Tabellini (1996a, 1996b) study risk sharing arrangements in a political economy setting with a local and a national government. The local policy redistributes across individuals and affects the probability of aggregate shocks, whereas the federal policy shares international risk. Their results indicate a trade off between federal risk sharing and moral hazard and federal risk sharing and redistribution, respectively. Both papers focus on political economy outcomes under alternative fiscal constitutions. Lockwood (1999) considers the fact that local public goods may give rise to spill over effects. He studies the central government's trade off between providing insurance and offering direct corrective incentives for local public goods. His

[3]Konrad (1994) presents a model where individuals know that a public good is to be privately provided in the future. This distorts effort incentives, because the individuals aim to reduce their disposable income to shift the burden of the public good provision to the other individuals. Similarly, Coate (1995) argues that unconditional transfers to the poor by the altruistic rich have negative efficiency effects. However, in our setting there is no altruism. By providing a federal insurance mechanism, the federal government sets the wrong incentives for the regions to choose the efficient self-insurance level. This leads to an inefficiently low level of self-insurance and to an inefficiently large loss.

approach focus on insurance through the provision of a public good, while in our setting, insurance itself is the public good.

Mansoorian (2000) considers risk sharing among individuals within and across regions in a federation with population mobility and infinite horizons. He finds that the regional authorities will not fully exploit gains from interregional risk sharing when population mobility is imperfect. In the Nash equilibrium there is complete risk sharing among the individuals within each region. Regional authorities who care on their reputation may be able to commit to an efficient allocation. Aronsson and Wikström (2003) analyze risk sharing arrangements in an optimal taxation framework. Two levels of government provide public goods and expenditures are financed via a labor income tax with a tax base being responsive to the private agent's labor supply decisions. The labor supply decisions as well as the choices of income tax rates are carried out under uncertainty because the localities may experience random productivity shocks. Part of the central government's decision problem is to provide tax revenue sharing between the local governments. The optimal degree of revenue sharing depends on whether or not the localities differ with respect to labor supply incentives.

In their seminal contribution, Ehrlich and Becker (1972) coined the terms self-insurance and self-protection for situations where the size of the loss and the probability of the loss can be influenced. Newer contributions in the insurance literature (Kunreuther and Heal (2003), Muermann and Kunreuther (2008), Lohse et al., forthcoming) and the public economics literature (Ihori and McGuire (2007), Ihori and McGuire, 2010) have extended the analysis of self-insurance and self-protection to the case where they are the outcome of a collective effort. In a recent paper, closest to ours, Goodspeed and Haughwout (2007) apply the Persson and Tabellini setting to examine the effects of natural disasters in a federation where local levels may influence the probability of the related losses in a self-protection manner. In line with Bordignon et al. (2001) they show that when the federal government is committed to full insurance against disasters, the local level has incentives to under invest in costly protective measures since the benefit of a reduction of the probability of the loss is shared by all. Compared to a first-best setting, second-best transfer levels (and the corresponding local investment levels) can be greater or smaller depending on the relative probability of a disaster. Our self-insurance model applies and extends Goodspeed and Haughwout's approach to a self-insurance situation.

The remainder of the paper is organized as follows. In the next section, we analyse a region's situation in case of autarky. In Section 3 we derive the first-best federation equilibrium where the central authority is able to implement and enforce an income transfer mechanism between the regions after the loss has been realized. However, in the second-best federation equilibrium in Section 4, the central government designs a second-best transfer scheme taking into account the regions' best-response behaviour to the announced transfer scheme. We consider both, variable and fixed transfers and provide a welfare analysis, too. Section 5 concludes.

2 The case of autarky

Consider two isolated regions indexed by $i = 1, 2$, each of them ruled by its own government.[4] In every period, region i earns from its citizens a constant tax revenue given by R. The regions are assumed to be symmetric in the sense that their tax revenue R is equal. This allows to focus entirely on insurance incentives and rules out redistribution motives. The time structure comprises two periods. In each period $j = 1, 2$ region i has income Y_{ij}, where the first index refers to the region $i = 1, 2$ and the second index stands for the period $j = 1, 2$.

In the first period, income is certain. Income in the second period is uncertain, because with probability p there occurs a loss L. Such a situation will be referred to as a loss situation L leading to low future income. In contrast, with probability $1 - p$ the regions end in a high income situation H with no loss. Crucially, each region may influence the size of the loss L by investing I_i (at a normalized price of one per unit of I) to reduce the size of the potential loss L in the second period, $L(I_i)$. We assume $I < L(I_i)$ to avoid a corner solution $I = 0$.

The income level in period 1 is given by

$$Y_{i1} = R - I_i, \qquad i = 1, 2. \tag{1}$$

Utility from this income is derived from a strictly monotonically increasing and strictly concave utility function $u(Y_{i1}) = u(R - I_i)$. The uncertain income in period 2 is

$$Y_{i2} = \begin{cases} R - L(I_i) & \text{with probability} \quad p, \\ R & \text{with probability} \quad 1 - p. \end{cases} \tag{2}$$

Utility from second period income stems from a strictly monotonically increasing and strictly concave utility function $v(\cdot)$.[5] The loss probabilities of the two regions are stochastically independent from each other.[6] Realistically, we assume that the loss is decreasing in the self-insurance investment with diminishing returns, e. g., $\partial L(I)/\partial I < 0$ and $\partial^2 L(I)/\partial I^2 > 0$. For the sake of simplicity and without loss of generality, we assume that there is no borrowing and no discounting.

In the case of isolated and independent regions, which we will call autarky, each regional government chooses its level of self-insurance investment I_i to maximize the sum of utilities in periods 1 and 2

$$\max_{I_i} U_i := u(R - I_i) + pv(R - L(I_i)) + (1 - p)v(R), \tag{3}$$

where the first summand is the first period utility, and the other two summands are the expected utility in the second period. U_i denotes the total intertemporal utility of region i.

[4]In the following, we will always speak of regions and of a central government, but our model applies to any pairing of hierarchical administrative institutions, e. g. countries and supra-national organization like the EU, or villages within larger regions. By considering only two entities, there are no problems of subgroup formations who may destabilize risk sharing arrangements as in Genicot and Ray (2003).

[5]By denoting second-period utility differently than the first-period one, we just want to avoid confusion in the analysis, but we do not make any assumption on different degrees of risk aversion etc.

[6]Here we make the usual assumption in the literature that the risks are not correlated across the regions. If they were, then insurance would lose much of its appeal since a loss would affect all regions more or less equally and there would be no true risk consolidation. Kunreuther and Heal (2003) analyze the case of interdependent risks in a static setting.

The condition describing implicitly the payoff maximising self-insurance level under autarky (denoted by superscript A) is[7]

$$\frac{\partial u(R - I_i^A)}{\partial Y_{i1}} \frac{\partial Y_{i1}}{\partial I_i} = -p \frac{\partial v(R - L(I_i^A))}{\partial Y_{i2}} \frac{\partial Y_{i2}}{\partial L} \frac{\partial L(I_i^A)}{\partial I_i}, \tag{4}$$

By the symmetry of the regions we obtain $I_1^A = I_2^A = I^A$. The left hand side (LHS) is the marginal cost of loss reduction in units of marginal utility in period one. The right hand side (RHS) gives the marginal benefit of loss reduction, i. e, the expected decrease of the loss, in units of marginal utility in period two. Hence, regional governments optimally choose the self-insurance effort to equalize probability adjusted marginal utilities over time. To allow an easy comparison with later conditions, we will use condition (4) rearranged as follows:

$$\frac{\frac{\partial u(R - I_i^A)}{\partial Y_{i1}} \cdot \frac{\partial Y_{i1}}{\partial I_i}}{\frac{\partial L(I_i^A)}{\partial I_i}} = -p \frac{\partial v(R - L(I_i^A))}{\partial Y_{i2}} \frac{\partial Y_{i2}}{\partial L}, \tag{5}$$

and prove the following lemma that will be used frequently in our analysis:

Lemma 1
The marginal utility cost of self-insurance, measured in units of marginal loss reduction, i. e., the LHS of equation (5), is an increasing function of self-insurance investment. The marginal benefit of loss reduction, measured in units of marginal utility in period two, i. e., the RHS of equation (5), is a decreasing function of self-insurance investment.

Proof. See Appendix.

Lemma 1 is also related to the concavity of the payoff function (3) in I. The latter proves that condition (5) describes a utility maximum.

3 The first-best federation equilibrium

3.1 The time structure

Suppose now that both regions are part of a broader unit or country ruled by a benevolent central government. This central authority is able to implement and enforce a income transfer mechanism between the regions after the loss has been realized. This redistribution scheme is described by transfers T_{MN}^i, $M, N \in (H, L)$. The superscript i denotes the region. The index MN denotes the state of region 1 (M) and region 2 (N), where M and N can both be equal to L (region has been hit by a loss and has a low income) and H (region is in a high income situation). We assume that the transfers are self financing and that the central government does not profit from its efforts, such that for a given outcome MN:

$$T_{MN}^1 + T_{MN}^2 = 0, \quad M, N \in \{H, L\}. \tag{6}$$

This assumptions means, in effect, that $T_{MN}^1 = -T_{MN}^2$ always holds. This allows us to drop the superscripts and to simplify our notation, since the transfer received by region M will always

[7] This payoff function is strictly concave in I, see Appendix.

equal the transfer paid by region N and viceversa. Without loss of generality, we will adopt the perspective of region 1, i. e., we will add the transfer to the payoff of region 1 and subtract it from the payoff of region 2. Of course, this is just a notation issue, since transfers can be either positive or negative.

The transfers T_{MN} can be fixed to a specific amount $\hat{T}$, or they can be conditional on the size of the loss L. This realization is not known yet when the transfer scheme is designed. When the transfer payments are calculated, the loss will have occurred or not, and so the size of the loss can be used to compute the transfer payments according to the central government's scheme.

The time structure of our model is as follows:

1. In the first stage, the central government designs and commits to a redistribution scheme for the regions.

2. In the second stage, regional governments choose an investment level I_1 and I_2 taking into account the central government's announced scheme.

3. In the third and final stage, income is realized and the regions have suffered a loss or not. Transfers are realized according to the scheme chosen in Stage 1.

If the central government implements a redistribution scheme, income in the second period for region 1 is given as

$$Y_{12} = \begin{cases} R - L(I_1) + T_{LL} & \text{with probability} \quad p^2, \\ R - L(I_1) + T_{LH} & \text{with probability} \quad p(1-p), \\ R + T_{HL} & \text{with probability} \quad (1-p)p, \\ R + T_{HH} & \text{with probability} \quad (1-p)^2, \end{cases} \tag{7}$$

and for region 2:

$$Y_{12} = \begin{cases} R - L(I_2) - T_{LL} & \text{with probability} \quad p^2, \\ R - L(I_2) - T_{HL} & \text{with probability} \quad (1-p)p, \\ R - T_{LH} & \text{with probability} \quad p(1-p), \\ R - T_{HH} & \text{with probability} \quad (1-p)^2, \end{cases} \tag{8}$$

We assume throughout the paper that the self-insurance investment levels I_1 and I_2 are not observable. If the self-insurance investment I was observable, a central government aiming to implement an investment level $\hat{I}$ could simply setup the following transfer scheme:

$$T = \begin{cases} \hat{T} \text{ for } \hat{I} \\ 0 \text{ else} \end{cases} \tag{9}$$

and in effect coerce the regions to choose $\hat{I}$.[8]

[8]While one may argue that self-insurance may be observable, for our results it suffices to assume that the investment level I is not verifiable, so the central government cannot implement a transfer scheme directly dependent on the self-insurance level. We believe this assumption to be quite realistic in practice.

3.2 First-best transfers

Let us analyze first as a benchmark case the case of an economic federation with a benevolent central government that has the power to choose both the transfer scheme and the self-insurance effort levels of the regions. Alternatively, such a scenario resembles a centralized country in which the regional government has no decision power. For this first-best outcome where the central government is able to dictate both the transfer scheme and the self-insurance effort choices, we can analyze the decisions one after the other as follows. Since the central government controls every decision, it does not matter whether the regional transfers are fixed or conditional on the size of the loss. The benevolent central government will always maximize the joint welfare of its regions.

The central government's first-best welfare maximization problem with respect to the efficient transfers T_{MN} is:

$$\begin{aligned}\max_{T_{MN}{}^{M,N\in\{H,L\}}} \quad W = U_1 + U_2 = u(R - I_1) + u(R - I_2) \\ + p^2(v(R - L(I_1) + T_{LL}) + v(R - L(I_2) - T_{LL})) \\ + (1-p)^2(v(R + T_{HH}) + v(R - T_{HH})) \\ + (1-p)p\,(v(R + T_{HL}) + v(R - L(I_2) - T_{HL})) \\ + p(1-p)\,(v(R - L(I_1) + T_{LH}) + v(R - T_{LH}))\,,\end{aligned} \tag{10}$$

where we have already used that transfers are self-funding. The first order conditions are:[9]

$$\frac{\partial v(R - L(I_1) + T_{LL})}{\partial Y_{12}} = \frac{\partial v(R - L(I_2) - T_{LL})}{\partial Y_{22}}, \tag{11}$$

$$\frac{\partial v(R + T_{HH})}{\partial Y_{12}} = \frac{\partial v(R - T_{HH})}{\partial Y_{22}}, \tag{12}$$

$$\frac{\partial v(R + T_{HL})}{\partial Y_{12}} = \frac{\partial v(R - L(I_2) - T_{HL})}{\partial Y_{22}}, \tag{13}$$

$$\frac{\partial v(R - L(I_1) + T_{LH})}{\partial Y_{12}} = \frac{\partial v(R - T_{LH})}{\partial Y_{22}}. \tag{14}$$

Transfers are chosen such that second-period marginal utilities are equalized. This requires that second-period income is identical across regions, implying a complete income equalization. If none of the regions has suffered a loss, there will be nothing to redistribute and transfers will be zero, $T_{HH} = 0$. In the case only one region is affected by a loss, transfers exhibit a transfer from the region H that is not affected by a loss to the region L being affected by the loss. Full risk-sharing leads to $R + T_{HL} = R - L(I_2) - T_{HL}$ and $R - L(I_1) + T_{LH} = R - T_{LH}$, respectively. It follows that $T_{HL} = -\frac{1}{2}L(I_2)$ and $T_{LH} = \frac{1}{2}L(I_1)$. If both regions have experienced a loss, there may be a transfer if the regions have chosen different self-insurance levels. However, we will see below that

[9] The second order condition holds because the cross partial derivatives are zero and the Hessian matrix is a diagonal matrix. The diagonal elements are the second order derivatives of the objective function (10) with respect to T_{LL}, T_{HH}, T_{HL}, and T_{LH}, which are negative by the concavity of u and v. Thus the Hessian matrix is negative definite.

the first-best entails equal self-insurance investment levels also leading to zero transfers, $T_{LL} = 0$, for this case.

The central government sets the first-best regional government's investment level in loss reduction $I_i, i = 1, 2$ by maximizing the joint payoff of the regions, for given ex-post equalization of income levels:

$$\begin{aligned} \max_{I_1, I_2} W = & \, u(R - I_1) + u(R - I_2) \\ & + p^2(v(R - L(I_1) + T_{LL}) + v(R - L(I_2) - T_{LL})) \\ & + (1-p)^2(v(R) + v(R)) \\ & + (1-p)p2v(R - \frac{1}{2}L(I_2)) \\ & + p(1-p)2v(R - \frac{1}{2}L(I_1)) \end{aligned} \tag{15}$$

where we have already inserted first-best income equalizing transfers. The first order conditions with respect to I_i, $i = 1, 2$ are[10]

$$\frac{\frac{\partial u(R - I_1)}{\partial Y_{11}} \frac{\partial Y_{11}}{\partial I_1}}{\frac{\partial L(I_1)}{\partial I_1}} = -p^2 \frac{\partial v(R - L(I_1) + T_{LL})}{\partial Y_{12}} \frac{\partial Y_{12}}{\partial L} - p(1-p) \frac{\partial v(R - \frac{1}{2}L(I_1))}{\partial Y_{12}} \frac{\partial Y_{12}}{\partial L} \tag{16}$$

$$\frac{\frac{\partial u(R - I_2)}{\partial Y_{21}} \frac{\partial Y_{21}}{\partial I_2}}{\frac{\partial L(I_2)}{\partial I_2}} = -p^2 \frac{\partial v(R - L(I_2) - T_{LL})}{\partial Y_{22}} \frac{\partial Y_{22}}{\partial L} - p(1-p) \frac{\partial v(R - \frac{1}{2}L(I_2))}{\partial Y_{22}} \frac{\partial Y_{22}}{\partial L} \tag{17}$$

Conditions (16) and (17) are symmetric. It follows that $I_1 = I_2$, and $T_{LL} = 0$ is a solution to the maximization problem. For $I_1 = I_2$, conditions (16) and (17) are identical and collapse to the single condition

$$\frac{\frac{\partial u(R - I^*)}{\partial Y_{i1}} \frac{\partial Y_{i1}}{\partial I}}{\frac{\partial L(I^*)}{\partial I}} = -p^2 \frac{\partial v(R - L(I^*))}{\partial Y_{i2}} \frac{\partial Y_{i2}}{\partial L} - p(1-p) \frac{\partial v(R - \frac{1}{2}L(I^*))}{\partial Y_{i2}} \frac{\partial Y_{i2}}{\partial L}. \tag{18}$$

which implicitly defines the first-best level of self-insurance I^*. To sum up, $I_1 = I_2 = I^*$ and $T_{LL} = 0$, $T_{HL} = -T_{LH} = T^* = -\frac{1}{2}L(I^*)$ are a solution to the first-best problem. It is the only solution, because the objective function (15) is strictly concave and continuously differentiable, so the optimization problem has only one local maximum. This local maximum is also the global maximum because $T_{LL} = 0$ and $T_{HH} = 0$ are already at the boundary and because T_{HL} and T_{LH} cannot be at the border given that the autarky solution is an interior solution.

The LHS of equation (18) represents the marginal cost of self-insurance due to an income decrease in the first period. The RHS displays the marginal benefit from such self-insurance, given by the marginal expected increase in period 2 utility. In contrast to the case of autarky,

[10] Again, the Hessian matrix is a diagonal, negative definite matrix, see Appendix, so the payoff function is concave in I_1 and I_2.

the presence of the transfers has inserted an externality because now the one region's investment affects the utility of the other region. Investments in self-insurance in region i also benefit region j ($i \neq j$), because the utility and income equalizing transfer is smaller. This marginal benefit only accrues in the situation where one region suffers a loss and the other region is spared. Therefore, the positive externality is weighted with the probability $p(1-p)$ for this case. The first-best transfer $T^* = -1/2L(I^*)$ ensures that the ex-post income levels are equalized.

To compare the self-insurance effort level under autarky with the first-best level in a federation, we have to compare the self-insurance levels I^A and I^* resulting from conditions (5) and (18), for which we will use the shorthand notation $LHS^A(I^A) = RHS^A(I^A)$ and $LHS^*(I^*) = RHS^*(I^*)$, respectively.

Proposition 1 (First-best federation equilibrium)
The first-best self-insurance level in a federation is smaller than the efficient level under autarky,

$$I^* < I^A, \tag{19}$$

and a region's utility in a federation is higher than under autarky,

$$U^* > U^A, \tag{20}$$

where U^* and U^A denote the intertemporal utility of each individual region in a first-best federation situation and under autarky, respectively.

Proof. To compare the self-insurance effort level under autarky with the first-best level in a federation, we have to compare the self-insurance levels I^A and I^* resulting from conditions (5) and (18). The LHS of both conditions are equal, so any difference in I will result from the RHS. For a given self-insurance level $\hat{I}$, the RHS^* is smaller than the RHS^A, since the transfer T^* increases the argument and decreases marginal utility in the case where one region is hit by a loss and the other region is not. It follows that the LHS must be smaller, too, which by Lemma 1 means that the self-insurance investment level I is smaller. Thus, the I^* that solves condition (18) is smaller than the I^A that solves condition (5). To compare the utility levels, consider the levels under autarky U^A with first-best self-insurance level in a federation U^*:

$$U^A(I^A) = u(R - I^A) + pv(R - L(I^A)) + (1-p)v(R) \tag{21}$$

$$U^*(I^*) = u(R - I^*) + p^2v(R - L(I^*)) + (1-p)^2v(R) + 2(1-p)pv(R - \frac{1}{2}L(I^*)). \tag{22}$$

It follows that

$$\begin{aligned} U^A(I^A) &< u(R - I^A) + p\left(pv(R - L(I^A)) + (1-p)v(R - \frac{1}{2}L(I^A))\right) \\ &\quad + (1-p)\left((1-p)v(R) + pv(R - \frac{1}{2}L(I^A))\right) \\ &= u(R - I^A) + p^2v(R - L((I^A))) + (1-p)^2v(R) + 2(1-p)pv(R - \frac{1}{2}L(I^A)) \\ &= U^*(I^A) < U^*(I^*), \end{aligned} \tag{23}$$

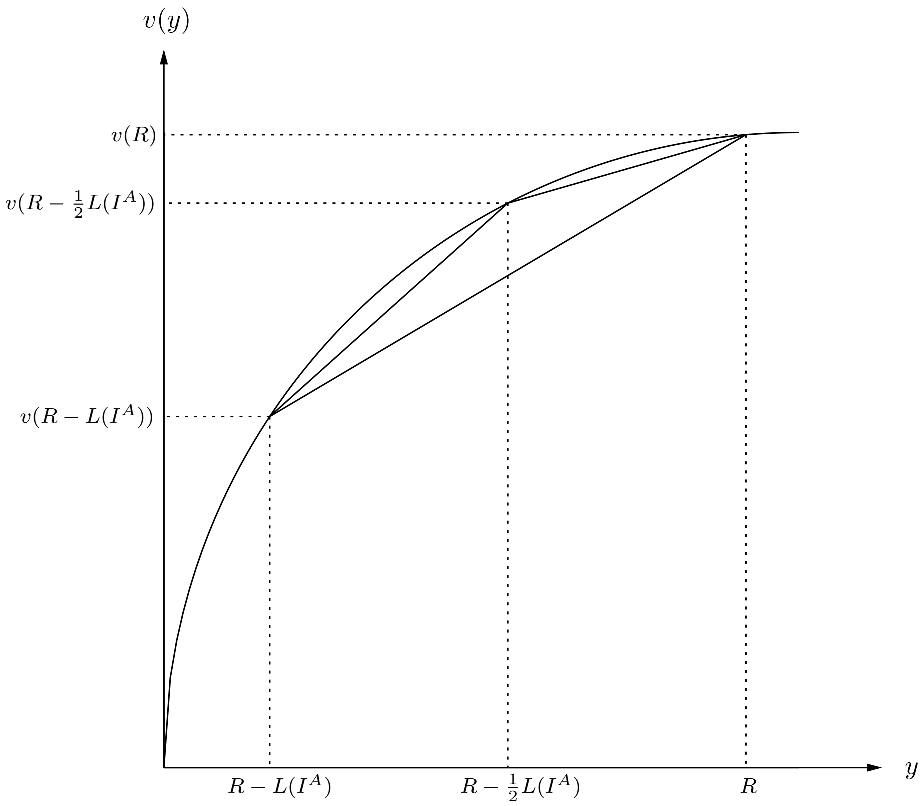

Figure 1: The concavity of the payoff function ensures that the first best payoff in a federation is higher than under autarky (insurance effect).

where the first inequality follows by the concavity of v (see Figure 1) and the second inequality follows from the fact that I^* maximizes W which given the definition of W in (15) implies I^* maximizes U^*. QED.

The intuition behind Proposition 1 is clear. In autarky, a region earns the full marginal benefit of its self-insurance effort. In contrast, in a federation, with a probability $p(1-p)$ one region is affected by a loss and the other region is not, in which case the ex-post rich region pays a transfer to the ex-post poor region. This redistribution reduces the marginal benefit of self-insurance. The regions have an incentive to build a federation and to consolidate the risk, but this risk consolidation reduces the efficient level first-best level of self-insurance. However, their welfare level increases, since they have an additional instrument to consolidate the risk, namely the insurance in the federation.

3.3 First-best and non-cooperative regions for a variable transfer scheme

Consider now the situation where the central government is not able to impose a self-insurance level on the regions and that the two regions act non-cooperatively for a given transfer scheme announced by the the central government. Is the first-best outcome feasible? Each region maximizes its own regional payoff in a non-cooperative way, taking the central governments' transfer scheme and the behaviour of the other region as given (Nash behaviour). First we look at the case where the central government has chosen a variable transfer conditional on the size of the loss. The first-best transfer is $T_{LH} = 1/2L(I_1)$ and $T_{HL} = -1/2L(I_2)$ if one of the regions suffers a loss and the other does not. If both regions end up in the same state, transfers are zero, $T_{HH} = T_{LL} = 0$. This transfer scheme is taken into account by region 1 when calculating its payoff

$$\begin{aligned} U_1^* &= u(R - I_1) + p^2 v(R - L(I_1)) \\ &\quad + p(1-p)v(R - 1/2L(I_1)) + (1-p)pv(R - 1/2L(I_2)) + (1-p)^2 v(R). \end{aligned} \tag{24}$$

Region 2's payoff is analogous. Since both regions are symmetric and the transfer scheme is also symmetric, the interior payoff maximising solution is identical for both regions. It follows that both regions will choose the same self-insurance effort and will have the same income before the possible realization of the loss. If both regions end up in the same state, transfers are zero, $T_{HH} = T_{LL} = 0$. Consider without loss of generality region 1. It chooses the investment level to maximise the payoff given by (24). The FOC implicitly describing the non-cooperative choice I_1^N is:

$$\frac{\frac{\partial u(R - I_1^N)}{\partial Y_{11}} \frac{\partial Y_{11}}{\partial I_1}}{\frac{\partial L(I^N)}{\partial I_1}} = -p^2 \frac{\partial v(R - L(I_1^N))}{\partial Y_{12}} \frac{\partial Y_{12}}{\partial L} - \frac{1}{2} p(1-p) \frac{\partial v(R - 1/2L(I_1^N))}{\partial Y_{12}} \frac{\partial Y_{12}}{\partial L}, \tag{25}$$

where the superscript N denotes the self-insurance effort in the non-cooperative Nash behaviour setting. The second-order conditions hold again due to the concavity of u and v and the assumptions about $L(I)$.

The structure of condition (25) is similar to the structure of the first-best condition (18) and the terms can be interpreted in a similar way. However, when choosing the self-insurance effort level, region 1 only takes into account the effect on its utility in stage 2 and disregards the positive externality on region 2. Therefore, the marginal benefit for the situation when there is an income transfer (for which the probability is $p(1-p)$) is half as big as in the first-best situation.

Proposition 2 (Free-riding incentive)
Suppose the central government designs a transfer scheme with a variable transfer that depends on the loss and that equalizes ex-post income levels. If the regions choose their self-insurance effort non-cooperatively in a Nash way, both regions choose the same self-insurance level $I_1^N(T^*) = I_2^N(T^*) = I^N(T^*)$ which is smaller than the first-best level I^* in a federation:

$$I^N(T^*) < I^*. \tag{26}$$

Proof. To establish (26), we compare the non-cooperative self-insurance level I^N with the first-best level I^*, and thus use conditions (25) and (18). The $RHS^N(I)$ for the non-cooperative equilibrium with first-best transfers (25) is smaller than the $RHS^*(I)$ for the first-best condition (18), $RHS^N(I) < RHS^*(I)$, because the former non-cooperative condition does not include the positive externality of the self-insurance investment. In an analogous way to the proof of Proposition 1 it follows that $I^N(T^*) < I^*$. QED.

The intuition is that in the second stage the regional governments take into account the transfers set by the central government when they take the non-cooperative decision about how much to invest in the first period in loss reduction and, consequently, invest too little in self-insurance effort in Stage 2. If the self-insurance effort levels I_1 and I_2 are not observable and not verifiable and/or the central government has no power to coordinate the regional governments towards the first-best outcome, the first-best outcome is not feasible. The regions may be sovereign countries and the central government may be a supranational organization. It follows that the central government takes into account the behaviour of the regions and when designing a a second-best transfer scheme in Stage 1.

4 The second-best federation equilibrium

As shown in the previous section, the first-best equilibrium is not feasible if regions act non-cooperatively. The central government has to take into account such behavior when designing its transfers. The resulting situation is a second-best setting.

4.1 The second-best transfers

We solve the game backwards and start from Stage 2, the last stage where an action is taken. Without loss of generality we consider region 1. Its payoff maximising non-cooperative choice for a given transfer scheme is:

$$\begin{aligned} U_1 &= u(R - I_1) + p^2 v(R - L(I_1)) \\ &\quad + p(1-p)v(R - L(I_1) + T_{LH}) + (1-p)pv(R - T_{HL})) + (1-p)^2 v(R). \end{aligned} \tag{27}$$

It chooses the investment level I_1 to maximise the payoff given by (27). The FOC implicitly describing the non-cooperative choice I_1^N is:

$$\frac{\dfrac{\partial u(R-I_1^N)}{\partial Y_{11}}\dfrac{\partial Y_{11}}{\partial I_1}}{\dfrac{\partial L(I^N)}{\partial I_1}} + p^2 \frac{\partial v(R - L(I_1^N))}{\partial Y_{12}}\frac{\partial Y_{12}}{\partial L} + p(1-p)\frac{\partial v(R - L(I_1^N) + T_{LH})}{\partial Y_{12}}\frac{\partial Y_{12}}{\partial L} = 0, \tag{28}$$

where the superscript N denotes the self-insurance effort in the non-cooperative Nash behaviour setting. The second-order conditions hold again due to the concavity of u and v and the assumptions about $L(I)$. This FOC defines a reaction function $I_1^N(T_{LH})$ for region 1. Lemma 2 shows that this reaction function has a strictly negative slope.

Lemma 2

Consider a Nash non-cooperative setting where one region suffers a loss and the other does not. If the central government increases (in absolute terms) the ex-post transfer from the richer, no loss region to the poorer, loss affected region, the regions have an incentive to decrease their investment in self-insurance, i. e.

$$\frac{\partial I_1^N}{\partial T_{LH}} < 0 \qquad \textit{and} \qquad \frac{\partial I_2^N}{\partial T_{HL}} > 0. \tag{29}$$

Proof. See Appendix.

These reaction functions from Lemma 2 represent the individual non-cooperative response of the regions for a given transfer scheme of the central government. Intuitively, the self-insurance efforts in a non-cooperative situation decrease if the ex-post transfers increase because self-insurance is a public good and the regions are contributing privately to it. In such contributions games, the individual agents do not take into account the positive externality produced upon the other players and thus reduce their effort compared with the first-best level.

In Stage 1, the central government designs a transfer scheme to maximize the joint payoff of the regions, taking into account the regions' reaction functions $I_1^N(T_{LH})$ and $I_2^N(T_{HL})$. The second-best maximization problem

$$\begin{aligned}\max_{T_{LH},T_{HL}} W^{SB} &= u(R - I_1^N(T_{LH})) + u(R - I_2^N(T_{HL})) \\ &+p^2 \left(v(R - L(I_1^N(T_{LH}))) + v(R - L(I_2^N(T_{HL})))\right) + (1-p)^2 2v(R) \\ &+(1-p)p \left(v(R + T_{HL}) + v(R - L(I_2^N(T_{HL})) - T_{HL})\right) \\ &+p(1-p) \left(v(R - L(I_1^N(T_{LH})) + T_{LH}) + v(R - T_{LH})\right).\end{aligned} \tag{30}$$

has the first order conditions with respect to T_{LH} (SB denotes second-best)

$$\begin{aligned}p(1-p)\frac{\partial v(R - T_{LH}^{SB})}{\partial Y_{22}} &= p(1-p)\frac{\partial v(R - L(I_1^N(T_{LH}^{SB})) + T_{LH}^{SB})}{\partial Y_{12}} \\ &+\frac{\partial I_1^N}{\partial T_{LH}} \cdot \left[\frac{\partial u(R - I_1^N(T_{LH}^{SB}))}{\partial Y_{11}}\frac{\partial Y_{11}}{\partial I_1^N}\right. \\ &+p^2\frac{\partial v(R - L(I_1^N(T_{LH}^{SB})))}{\partial Y_{12}}\frac{\partial Y_{12}}{\partial L} \cdot \frac{\partial L}{\partial I_1^N} \\ &\left.+p(1-p)\frac{\partial v(R - L(I_1^N(T_{LH}^{SB})) + T_{LH}^{SB})}{\partial Y_{12}}\frac{\partial Y_{12}}{\partial L} \cdot \frac{\partial L}{\partial I_1^N}\right]\end{aligned} \tag{31}$$

An analogous symmetric condition obtains for the derivative with respect to T_{HL}, which we omit for the sake of brevity.[11] The last three lines contain the individual maximization of a region and are equal to zero following FOC (28), such that (32) simplifies to[12]

$$p(1-p)\frac{\partial v(R - T_{LH}^{SB})}{\partial Y_{22}} = p(1-p)\frac{\partial v(R - L(I_1^N(T_{LH}^{SB})) + T_{LH}^{SB})}{\partial Y_{12}} \tag{32}$$

Proposition 3 (Second-best federation equilibrium)
If the central government designs a second-best transfer scheme with variable transfers taking into account the regions' best-response behavior to the anounced transfer scheme,

1. *the second-best transfers are always strictly greater in absolute terms than the corresponding first-best transfers,*

$$T_{LH}^{SB} = -T_{HL}^{SB} =: T^{SB} > T^*,$$

2. *the self-insurance level in the second-best non-cooperative setting is smaller than in the first-best situation*

$$I^N(T^{SB}) < I^*,$$

[11] The second order condition holds if W^{SB} is a concave function in T_{LH} and T_{HL}. A sufficient condition for concavity of the second best maximization problem is $\frac{\partial L}{\partial I_1^N} \cdot \frac{\partial I_1^N}{\partial T_{LH}} < 1$, which holds in the present model, see Appendix.

[12] Alternatively, we could have applied the envelope theorem to obtain (32) directly.

3. *and the second-best transfers are equal in relative terms to the corresponding first-best transfers, i. e. in both situations the transfers are equal to half the loss:*

$$T^{SB} = \frac{1}{2}L(I^N(T^{SB})).$$

Proof. To proof the first part, consider the simplified second best condition (32), which we repeat here for the sake of clarity and to introduce the notation LHS^{SB} and RHS^{SB}:

$$LHS^{SB} := \frac{\partial v(R - T^{SB})}{\partial Y_{22}} = \frac{\partial v(R - L(I_1^N(T^{SB})) + T^{SB})}{\partial Y_{12}} =: RHS^{SB}. \tag{33}$$

LHS^{SB} is increasing in T^{SB}, while RHS^{SB} is decreasing in T^{SB}. This second result follows from $\frac{\partial I_1^N}{\partial T_{LH}} \cdot \frac{\partial L}{\partial I_1^N} < 1$ (see Appendix). The corresponding FOC for the first best transfer T^* is condition (14):

$$LHS^* := \frac{\partial v(R - T^*)}{\partial Y_{22}} = \frac{\partial v(R - L(I^*) + T^*)}{\partial Y_{12}} =: RHS^*. \tag{34}$$

Now substitute T^* for T^{SB} in RHS^{SB} and let us compare the resulting $RHS^{SB}(T^*)$ to $RHS^*(T^*)$. By Proposition 2, $I^N(T^*) < I^*$. such that $L(I^N(T^*)) > L(I^*)$. The loss becomes larger and the argument smaller, so marginal utility increases and we obtain $RHS^*(T^*) < RHS^{SB}(T^*)$. Suppose now $T^{SB} < T^*$. Then it follows

$$LHS^*(T^*) = RHS^*(T^*) < RHS^{SB}(T^*) < RHS^{SB}(T^{SB}) = LHS^{SB}(T^{SB}) = LHS^*(T^{SB}). \tag{35}$$

The first inequality obtains from the result above. The second inequality follows from our assumption $T^{SB} < T^*$ and from the fact that RHS^{SB} is decreasing in T since $\frac{\partial L}{\partial I_1^N} \cdot \frac{\partial I_1^N}{\partial T_{LH}} < 1$. The inequality chain above means

$$LHS^*(T^*) < LHS^*(T^{SB}), \tag{36}$$

which is equivalent to $T^* < T^{SB}$, since the LHS of both first best and second best conditions are increasing in the transfer T. This contradicts the assumption $T^{SB} < T^*$ and proves $T^{SB} > T^*$.

The second part follows from $I_1^N(T^{SB}) < I_1^N(T^*) < I^*$, where the first inequality is due to the fact that $T^{SB} > T^*$ by part 1 and that I^N is decreasing in T by Lemma 2, while the second inequality is due to Proposition 2.

To show the third and last part of the proposition, consider again the FOC (32). By the concavity of the maximization program, if the marginal utilities in LHS^{SB} and RHS^{SB} are equal, the arguments must be equal, too:

$$R - T_{LH}^{SB} = R - L(I_1^N(T_{LH}^{SB})) + T_{LH}^{SB} \tag{37}$$

Rearranging leads to the third part of the proposition. QED.

Since the loss is always greater, the insurance transfers are also larger. Ex post, the central government always aims to equalize income levels across regions and it has no instrument to incentivate the individual regions to increase their self-insurance efforts I_N^{SB}, which depend negatively on the transfer level. Although the second-best transfers are equal to the corresponding first-best transfers in relative terms (i. e., relative to the realized loss), $T^{SB} = \frac{1}{2}L(I^N(T^{SB}))$, due to the insurance pooling in the federation the second-best self-insurance levels are always smaller in absolute terms than in the first-best situation: $I^N(T^{SB}) < I^*$.

4.2 Welfare analysis

A regions' utility level in the second-best outcome is

$$U^{SB}(I^N(T^{SB})) = u(R-I^N(T^{SB}))+p^2v(R-L((I^N(T^{SB}))+(1-p)^2v(R)+2(1-p)pv(R-\frac{1}{2}L(I^N(T^{SB}))). \tag{38}$$

This utility level is always strictly smaller than the first-best level in a federation, since according to Proposition 3, the second-best self-insurance effort is smaller than the first-best, $I^{SB} = I^N(T^{SB}) < I^*$. By Proposition 1, this also means that the self-insurance effect is smaller than in autarky, $I^N(T^{SB}) < I^A$. It remains to establish whether the regions' payoff is smaller in the second-best situation or under autarky. Two effects are at work. Under autarky, each region chooses the individually efficient self-insurance level. However, there is no pooling of the risk like in a federation. Being risk averse, this non-pooled risk carries a greater risk premium than in a federation. Joining a federation means that the risks are pooled (positive effect). However, it introduces a Samaritan's dilemma kind of effect and leads the regions to reduce their self-insurance effort (negative effect). It turns out that, depending on the parameters, each effect may dominate. In other words, joining a federation may increase, but also decrease the individual region's welfare.

Proposition 4 (Utility in the second-best federation equilibrium)
Utility of a region in the second-best federation equilibrium can be smaller or greater than welfare under autarky:

$$U^{SB}(I^N(T^{SB})) \gtreqless U^A(I^A), \tag{39}$$

depending on the preferences of the regions and specially depending on the probability p of the loss outcome.

Proof. Remember from (21) that $U^A(I^A) = u(R-I^A)+pv(R-L(I^A))+(1-p)v(R)$. It suffices to show by construction that depending on the probability p, sometimes welfare is higher under a second-best federation and sometimes under autarky. Suppose $p = 0$. Then we obtain

$$U^{SB}(I^{SB}) = u(R - I^{SB}) + v(R) > u(R - I^A) + v(R) = U^A(I^A), \tag{40}$$

because $I^{SB} < I^A$. By continuity, there exists an $\epsilon > 0$ such that $p = \epsilon$ and the inequality (40) still holds. Suppose now $p = 1$. It follows

$$U^{SB}(I^{SB}) = u(R - I^{SB}) + v(R - L((I^{SB}) < u(R - I^A) + v(R - L(I^A)) = U^A(I^A) \tag{41}$$

$$\iff \quad u(R - I^{SB}) - u(R - I^A) < v(R - L(I^A)) - v(R - L((I^{SB}) \tag{42}$$

$$(R - I^{SB} - R + I^A) \cdot u'(\hat{y}) < (R - L(I^A) - R + L(I^{SB})) \cdot v'(\tilde{y}) \tag{43}$$

$$\text{with} \quad \hat{y} \in [R - I^{SB}, R - I^A] \text{ and } \tilde{y} \in [R - L(I^A), R - L(I^{SB})]$$

$$\iff \quad (I^A - I^{SB}) \cdot u'(\hat{y}) < (L(I^{SB}) - L(I^A)) \cdot v'(\tilde{y}), \tag{44}$$

where the step with $\hat{y}$ and $\tilde{y}$ follows from the Mean Value Theorem and the last inequality holds because $I^A - I^{SB} < L(I^{SB}) - L(I^A)$ by our assumptions regarding the loss function L and $u'(\hat{y}) < u'(\tilde{y})$ by the concavity of the payoff functions $u(\cdot) = v(\cdot)$. Figure 2 illustrates this second case, the

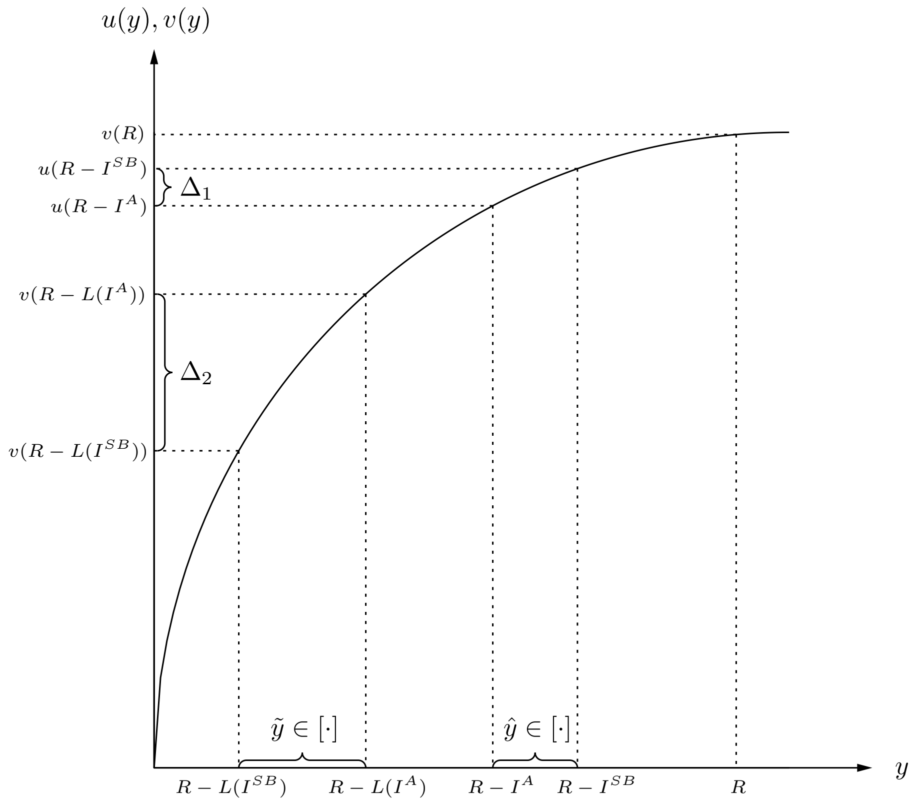

Figure 2: If the loss is sufficiently likely, the concavity of the payoff function ensures that the payoff in a federation is higher than under autarky, i. e. that the insurance effect dominates the disincentive effect.

distance Δ_1 is always smaller than Δ_2. Again, by continuity, there exists an $\epsilon > 0$ such that for $p = 1 - \epsilon$ the inequality (41) still holds. This proves that welfare can be higher under autarky or in the second best equilibrium. QED.

The ambiguity of Proposition 4 reflects the countervailing effects at work. Under autarky, each region has the right incentive to choose the efficient self-insurance level. Under a federation, the risk consolidation has a positive effect, but this risk reduction causes a Samaritan's dilemma and decreases the incentive to spend the first-best level of self-insurance effort. It depends on the specific parameters of the payoff function and on the probability distribution of the loss whether the autarky or the federation setting lead to a higher payoff for the regions.

4.3 Policy implications

Until now, we have assumed a variable transfer in the second-best setting. In the first-best full information situation, it does not matter whether the central government implements a variable transfer (which varies with the loss to equalize ex-post income levels across the regions) or whether the central government implements a fixed transfer, which in the following we will denote with an upper bar. Both transfers lead to the same first-best self-insurance effort levels. However, in a second-best non-cooperative setting, the design of the transfer scheme has a big effect and fixed and variable transfers do not lead to the same second-best outcome.

The previous sections have analysed the case of a variable transfer. Suppose that the central government has chosen a fixed transfer given by $\overline{T}_{HL} = -\overline{T}_{LH} = \overline{T}^* = -\frac{1}{2}L(I^*)$. Then, the payoff for region 1 is

$$\begin{aligned} U_1^* &= u(R - I_1) + p^2 v(R - L(I_1)) \\ &\quad + p(1-p)v(R - L(I_1) + \overline{T}^*_{LH}) + (1-p)pv(R + \overline{T}^*_{HL}) + (1-p)^2 v(R). \end{aligned} \tag{45}$$

Region 1 chooses the investment level I_1 to maximise the payoff (45). The region's FOC implicitly describing the non-cooperative choice I_1^N is:

$$\frac{\frac{\partial u(R - I_1^N)}{\partial Y_{11}} \frac{\partial Y_{11}}{\partial I_1}}{\frac{\partial L(I^N)}{\partial I_1}} = -p^2 \frac{\partial v(R - L(I_1^N))}{\partial Y_{12}} \frac{\partial Y_{12}}{\partial L} - p(1-p) \frac{\partial v(R - L(I_1^N) + \frac{1}{2}L(I^*))}{\partial Y_{12}} \frac{\partial Y_{12}}{\partial L}, \tag{46}$$

Proposition 5
Consider a federation where the central government designs and implements a transfer scheme with fixed transfers given by the first-best transfer levels $\overline{T}_{HL} = -\overline{T}_{LH} = \overline{T}^* = -\frac{1}{2}L(I^*)$. If the regions choose their self-insurance effort non-cooperatively in a Nash way, both regions choose the same self-insurance level $I_1^N(\overline{T}^*) = I_2^N(\overline{T}^*) = I^N(\overline{T}^*)$ which coincides with the first-best level I^*:

$$I^N(\overline{T}^*) = I^*. \tag{47}$$

Proof. For $I_1^N = I^*$, the first-order conditions (18) and (46) are identical and are equal to zero at $I_1^N = I^*$. By the concavity of the objective function, this must be the only maximum of the function. QED.

Proposition 5 shows that implementing fixed transfers leads to the first-best outcome while variable transfers that depend on the size of the loss set the wrong incentives. As a policy implication, central governments should not aim to equalize income levels across regions, but establish fixed transfer payments. Consider the federal transfer payments across regions in Germany. By law, the aim of the transfers is to "equalize living standards" across the Bundesländer. Not surprisingly, the richer states criticise that the poorer states invest too little in their well-being and rely too much in the intra-federal transfers, since the receiving states get a higher transfer the poorer they are. As an alternative, consider Bill Clinton's welfare reform in the US in the 1990s. The states get a fixed block grant from the central government and thus have an incentive to invest the first-best effort level given the fixed transfer.

5 Conclusion

This paper analyzes the risk sharing aspect in a federation consisting of two regions. The regions can be hit by a uniform shock leading to losses that occur with an exogenous probability and in a stochastically independent way. To reduce the size of the loss, the regions can invest in a public good that serves as a self-insurance device, i.e. it reduces the size of the loss for both regions at the same time. First, as a benchmark case, we derive the optimal level of regional self-insurance effort in case of autarky. This situation is compared to a first-best federation equilibrium where the central government can determine regions' self-insurance effort as well as an equalizing transfer scheme. The first-best self-insurance level of such federation equilibrium turns out to be smaller than the level under autarky which shows the risk consolidation effect of the federation. However, in case of non-cooperative regions, the first-best outcome is not feasible any more and the regions have an incentive for free -riding. In case of a second-best setting with first the central government designing and committing to a transfer scheme and second the regions deciding about their self-insurance effort, in equilibrium second-best transfers are higher than first-best ones whereas the regions' self-insurance levels are comparatively lower.

Despite the widely held belief, that joining a federation increases a region's welfare, we show that this is not necessarily the case. The welfare increase due to pooling risks in a federation may be partially or even fully outweighed by a Samaritan's dilemma kind of effect that regions in a federation reduce their self-insurance effort. To overcome this dilemma, the central government should commit to fixed (rather than variable) transfers at the first-best level. This induces non-cooperatively behaving regions to choose in turn a self-insurance effort in the same size as the first-best.

Some caveats may apply. First, our analysis does not deal with time inconsistencies. The question whether changes in the timing of equalizing transfers to regions necessitates an adjustment in federal corrective policy is addressed by Köthenbürger (2007). Second, our setting focuses on public goods with self-insurance character. The related case of public goods that serve as a public self-protection device is analyzed by Goodspeed and Haughwout (2007).

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A Appendix

A.1 Concavity of payoff (3) under autarky

The FOC (4) with respect to I_i is

$$\frac{\partial u(R - I_i^A)}{\partial Y_{i1}} \cdot \underbrace{\frac{\partial Y_{i1}}{\partial I_i}}_{-1} + \frac{\partial v(R - L(I_i^A))}{\partial Y_{i2}} \cdot \underbrace{\frac{\partial Y_{i2}}{\partial L}}_{-1} \cdot p \cdot \frac{\partial L(I_i^A)}{\partial I_i} = 0 \tag{48}$$

$$\iff -\frac{\partial u(R - I_i^A)}{\partial Y_{i1}} - \frac{\partial v(R - L(I_i^A))}{\partial Y_{i2}} \cdot p \cdot \frac{\partial L(I_i^A)}{\partial I_i} = 0 \tag{49}$$

Taking the derivative with respect to I_i results in the following SOC, which is negative:

$$-\frac{\partial^2 u(R - I_i^A)}{\partial Y_{i1}^2} \cdot \underbrace{\frac{\partial Y_{i1}}{\partial I_i}}_{-1} - \frac{\partial^2 v(R - L(I_i^A))}{\partial Y_{i2}^2} \cdot \underbrace{\frac{\partial Y_{i2}}{\partial L}}_{-1} \cdot \left(\frac{\partial L(I_i^A)}{\partial I_i}\right)^2 p - \frac{\partial v(R - L(I_i^A))}{\partial Y_{i2}} p \frac{\partial^2 L(I_i^A)}{\partial I_i^2} \tag{50}$$

$$= \underbrace{\frac{\partial^2 u(R - I_i^A)}{\partial Y_{i1}^2}}_{-} + \underbrace{p \cdot \frac{\partial^2 v(R - L(I_i^A))}{\partial Y_{i2}^2} \left(\frac{\partial L(I_i^A)}{\partial I_i}\right)^2}_{-} \underbrace{-p \cdot \underbrace{\frac{\partial v(R - L(I_i^A))}{\partial Y_{i2}}}_{+} \cdot \underbrace{\frac{\partial^2 L(I_i^A)}{\partial I_i^2}}_{+}}_{-} \tag{51}$$

A.2 Proof of Lemma 1

Remember the FOC under autarky,

$$\frac{\frac{\partial u(R - I_i^A)}{\partial Y_{i1}} \cdot \frac{\partial Y_{i1}}{\partial I_i}}{\frac{\partial L(I_i^A)}{\partial I_i}} = -p \frac{\partial v(R - L(I_i^A))}{\partial Y_{i2}} \frac{\partial Y_{i2}}{\partial L}.$$

The derivative of the LHS of (5) with respect to I_i is (using $\frac{\partial Y_{i1}}{\partial I_i} = -1$ and $\frac{\partial Y_{i2}}{\partial L} = -1$):

$$\frac{\overbrace{\frac{\partial L(I_i^A)}{\partial I_i}}^{-} \cdot (-1) \cdot \overbrace{\frac{\partial^2 u(R - I_i^A)}{\partial Y_{i1}^2}}^{-} \cdot (-1) - (-1) \cdot \overbrace{\frac{\partial u(R - I_i^A)}{\partial Y_{i1}}}^{+} \cdot \overbrace{\frac{\partial^2 L(I_i^A)}{\partial I_i^2}}^{+}}{\left(\frac{\partial L(I_i^A)}{\partial I_i}\right)^2} > 0. \tag{52}$$

The derivative of the RHS of (5) is:

$$-p \cdot (-1) \cdot \underbrace{\frac{\partial^2 v(R - L(I_i^A))}{\partial Y_{i2}^2}}_{-} \cdot (-1) \cdot \underbrace{\frac{\partial L(I_i^A)}{\partial I_i^A}}_{-} < 0. \tag{53}$$

A.3 Concavity of first-best payoff (10) in a first-best setting

To calculate the Hessian matrix of the payoff function (10) we look at the derivatives with respect to the four variables T_{LL}, T_{HH}, T_{HL}, and T_{LH}. Consider first the derivatives with respect to T_{LL}

$$\frac{\partial(10)}{\partial T_{LL}} = p^2 \frac{\partial v(R - L(I_1) + T_{LL})}{\partial Y_{1L}} \cdot \underbrace{\frac{\partial Y_{1L}}{\partial T_{LL}}}_{+1} + p^2 \frac{\partial v(R - L(I_2) - T_{LL})}{\partial Y_{2L}} \cdot \underbrace{\frac{\partial Y_{2L}}{\partial T_{LL}}}_{-1} \tag{54}$$

$$= = p^2 \frac{\partial v(R - L(I_2) - T_{LL})}{\partial Y_{1L}} - p^2 \frac{\partial v(R - L(I^2) - T_{LL})}{\partial Y_{2L}}$$

$$\frac{\partial^2(10)}{\partial T_{LL}^2} = p^2 \cdot \frac{\partial^2 v(R - L(I_1) + T_{LL})}{\partial Y_{1L}^2} \cdot \underbrace{\frac{\partial Y_{1L}}{\partial T_{LL}}}_{+1} - p^2 \frac{\partial^2 v(R - L(I_2) - T_{LL})}{\partial Y_{2L}^2} \cdot \underbrace{\frac{\partial Y_{2L}}{\partial T_{LL}}}_{-1} \tag{55}$$

$$= p^2 \frac{\partial^2 v(R - L(I_1) + T_{LL})}{\partial Y_{1L}^2} + p^2 \frac{\partial^2 v(R - L(I_2) - T_{LL})}{\partial Y_{2L}^2} < 0$$

$$\frac{\partial^2(10)}{\partial T_{LL} \partial x} = 0, \quad \text{where } x \text{ means any other variable.} \tag{56}$$

An analogous result obtains for the variable T_{HH}: the second derivative is negative and the cross partial derivative is zero. Consider now the derivative with respect to T_{HL}:

$$\frac{\partial(10)}{\partial T_{HL}} = (1-p)p \frac{\partial v(R + T_{HL})}{\partial Y_{1H}} \cdot \underbrace{\frac{\partial Y_{1H}}{\partial T_{HL}}}_{1} + (1-p)p \frac{\partial v(R - L(I_2) - T_{HL})}{\partial Y_{2L}} \cdot \underbrace{\frac{\partial Y_{2L}}{T_{HL}}}_{-1} \tag{57}$$

$$= (1-p)p \frac{\partial v(R + T_{HL})}{\partial Y_{1H}} - (1-p)p \frac{\partial v(R - L(I_2) - T_{HL})}{\partial Y_{2L}} \tag{58}$$

$$\frac{\partial^2(10)}{\partial T_{HL}^2} = (1-p)p \frac{\partial^2 v(R + T_{HL})}{\partial Y_{1H}^2} + (1-p)p \frac{\partial v^2(R - L(I_2) - T_{HL})}{\partial Y_{2L}^2} < 0 \tag{59}$$

$$\frac{\partial^2(10)}{\partial T_{HL} \partial x} = 0, \quad \text{where again } x \text{ means any other variable.} \tag{60}$$

Again, we obtain for T_{LH} an analogous result with a negative second derivative is and a zero cross partial derivative. The Hessian matrix of the first-best maximization problem (10) is

$$\begin{pmatrix} \frac{\partial^2(10)}{\partial T_{LL}^2} & 0 & 0 & 0 \\ 0 & \frac{\partial^2(10)}{\partial T_{HH}^2} & 0 & 0 \\ 0 & 0 & \frac{\partial^2(10)}{\partial T_{HL}^2} & 0 \\ 0 & 0 & 0 & \frac{\partial^2(10)}{\partial T_{LH}^2} \end{pmatrix} \tag{61}$$

This is a diagonal matrix where all the elements in the diagonal are negative. Thus, the matrix is negative definite and the solution described by the FOCs is a local maximum.

In a similar way, the Hessian matrix of the payoff function (15) with respect to I_1 and I_2 is also a diagonal, negative definite matrix, since

$$\frac{\partial(16)}{\partial I_1} = \frac{\partial^2 u(\cdot)}{\partial Y_{11}^2}\left(\frac{\partial Y_{11}}{\partial L}\right)^2 + p^2\frac{\partial^2 v(\cdot)}{\partial Y_{12}^2}\left(\frac{\partial Y_{12}}{\partial L}\right)^2\left(\frac{\partial L}{\partial I_1}\right)^2 + p^2\frac{\partial v(\cdot)}{\partial Y_{12}}\frac{\partial Y_{12}}{\partial L}\frac{\partial^2 L}{\partial I_1^2} \tag{62}$$

$$+ p(1-p)\frac{\partial^2 v(\cdot)}{\partial Y_{12}^2}\frac{\partial Y_{12}}{\partial L}\left(-\frac{1}{2}\right)\left(\frac{\partial L}{\partial I_1}\right)^2 + p(1-p)\frac{\partial v(\cdot)}{\partial Y_{12}}\frac{\partial Y_{12}}{\partial L}\frac{\partial^2 L}{\partial I_1^2} < 0 \tag{63}$$

$$\frac{\partial(17)}{\partial I_2} = \frac{\partial^2 u(\cdot)}{\partial Y_{11}^2}\left(\frac{\partial Y_{11}}{\partial L}\right)^2 + p^2\frac{\partial^2 v(\cdot)}{\partial Y_{12}^2}\left(\frac{\partial Y_{12}}{\partial L}\right)^2\left(\frac{\partial L}{\partial I_2}\right)^2 + p^2\frac{\partial v(\cdot)}{\partial Y_{12}}\frac{\partial Y_{12}}{\partial L}\frac{\partial^2 L}{\partial I_2} \tag{64}$$

$$+ p(1-p)\frac{\partial^2 v(\cdot)}{\partial Y_{12}^2}\frac{\partial Y_{12}}{\partial L}\left(-\frac{1}{2}\right)\left(\frac{\partial L}{\partial I_2}\right)^2 + p(1-p)\frac{\partial v(\cdot)}{\partial Y_{12}}\frac{\partial Y_{12}}{\partial L}\frac{\partial^2 L}{\partial I_2^2} < 0 \tag{65}$$

$$\frac{\partial(16)}{\partial I_2} = \frac{\partial(17)}{\partial I_1} = 0 \tag{66}$$

This proves that the solutions to the FOC of (15) correspond to maxima of the payoff (10).

A.4 Proof of Lemma 2

To show the first result, consider the FOC corresponding to the maximization of (27),

$$\frac{\partial u(R-I_1^N)}{\partial Y_{11}}\frac{\partial Y_{11}}{\partial I_1} + p^2\frac{\partial v(R-L(I_1^N))}{\partial Y_{12}}\frac{\partial Y_{12}}{\partial L}\frac{\partial L(I^N)}{\partial I_1} + p(1-p)\frac{\partial v(R-L(I_1^N)+T_{LH})}{\partial Y_{12}}\frac{\partial Y_{12}}{\partial L}\frac{\partial L(I^N)}{\partial I_1} = 0, \tag{67}$$

Apply the implicit function theorem to obtain

$$\frac{\partial I_1^N}{\partial T_{LH}} = -\frac{\frac{\partial FOC(67)}{\partial T_{LH}}}{\frac{\partial FOC(67)}{\partial I_1^N}}. \tag{68}$$

The denominator is negative due to the concavity of the objective function, so the sign of the reaction function depends solely on the sign of $\frac{\partial FOC(67)}{\partial T_{LH}}$, which is negative since

$$p(1-p)\underbrace{\frac{\partial^2 v(R-L(I_1^N)+T_{LH})}{\partial Y_{12}^2}}_{-}\underbrace{\frac{\partial Y_{12}}{\partial L}}_{-}\underbrace{\frac{\partial L}{\partial I_1^N}}_{-}\underbrace{\frac{\partial Y_{12}}{\partial T_{LH}}}_{+} < 0. \tag{69}$$

This establishes $\frac{\partial I_1^N}{\partial T_{LH}} < 0$. The second result follows in an analogous way using the analogous reaction function $I_2^N(T_{HL})$ that describes the individual payoff maximising behaviour for region 2.[13] QED.

[13] Note that, since the transfers are defined from the perspective of region 1, a positive transfer to region 1 when it has been hit by a loss is given when $T_{LH} > 0$. Conversely, a positive transfer for region 2 arises when $T_{HL} < 0$. So both results mean that the self-insurance effort level decreases, the greater the absolute transfer to a region when it is the only region affected by a loss.

A.5 Concavity of payoff (30) in a second-best setting

The FOC of (30) with respect to T_{LH} is (32). The SOC of (30) with respect to T_{LH} is given by

$$p(1-p)\frac{\partial^2 v(R-L(I_1^N(T_{LH}^{SB}))+T_{LH}^{SB})}{\partial Y_{12}^2}\left(1-\frac{\partial L}{\partial I_1^N}\cdot\frac{\partial I_1^N}{\partial T_{LH}}\right)+p(1-p)\frac{\partial^2 v(R-T_{LH}^{SB})}{\partial Y_{22}^2},$$

which is negative if $1-\frac{\partial L}{\partial I_1^N}\cdot\frac{\partial I_1^N}{\partial T_{LH}}>0$. We can verify this using the results above:

$$\frac{\partial I_1^N}{\partial T_{LH}}\cdot\frac{\partial L}{\partial I_1^N}=-\frac{\frac{\partial FOC(67)}{\partial T_{LH}}}{\frac{\partial FOC(67)}{\partial I_1^N}}\cdot\frac{\partial L}{\partial I_1^N}$$

$$=-\frac{p(1-p)\frac{\partial^2 v(R-L(I_1^N)+T_{LH})}{\partial Y_{12}^2}\frac{\partial Y_{12}}{\partial L}\frac{\partial L}{\partial I_1^N}\frac{\partial Y_{12}}{\partial T_{LH}}}{\frac{\partial^2 u(R-I_1^N)}{\partial Y_{11}^2}+p^2\frac{\partial^2 v(R-L(I_1^N))}{\partial Y_{12}^2}\left(\frac{\partial L(I^N)}{\partial I_1}\right)^2-p^2\frac{\partial v(R-L(I_1^N))}{\partial Y_{12}}\frac{\partial^2 L(I^N)}{\partial I_1^2}+p(1-p)\frac{\partial^2 v(R-L(I_1^N)+T_{LH})}{\partial Y_{12}^2}\left(\frac{\partial L(I^N)}{\partial I_1}\right)^2-p(1-p)\frac{\partial v(R-L(I_1^N)+T_{LH})}{\partial Y_{12}}\frac{\partial^2 L(I^N)}{\partial I_1^2}}\cdot\frac{\partial L}{\partial I_1^N}$$

$$=\frac{\left(p(1-p)\frac{\partial^2 v(R-L(I_1^N)+T_{LH})}{\partial Y_{12}^2}\frac{\partial Y_{12}}{\partial L}\frac{\partial L}{\partial I_1^N}\frac{\partial Y_{12}}{\partial T_{LH}}\right)\frac{\partial L}{\partial I_1^N}}{-\frac{\partial^2 u(R-I_1^N)}{\partial Y_{11}^2}-p^2\frac{\partial^2 v(R-L(I_1^N))}{\partial Y_{12}^2}\left(\frac{\partial L(I^N)}{\partial I_1}\right)^2+p^2\frac{\partial v(R-L(I_1^N))}{\partial Y_{12}}\frac{\partial^2 L(I^N)}{\partial I_1^2}-p(1-p)\frac{\partial^2 v(R-L(I_1^N)+T_{LH})}{\partial Y_{12}^2}\left(\frac{\partial L(I^N)}{\partial I_1}\right)^2+p(1-p)\frac{\partial v(R-L(I_1^N)+T_{LH})}{\partial Y_{12}}\frac{\partial^2 L(I^N)}{\partial I_1^2}},$$

where $\frac{\partial I_1^N}{\partial T_{LH}}\cdot\frac{\partial L}{\partial I_1^N}\in(0,1)$ follows because the denominator is greater than the numerator.