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Endogenous Response to the 'Network Tax'*

José Pedro Fique†

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Keywords: Financial Network, Regulation, Counterparty Risk, Liquidity Coinsurance.

JEL Classification Numbers: D85, G18, G21.

1 Introduction

The turmoil in the financial markets that had its roots in the 2007 US subprime crisis prompted government action all over the world motivated by contagion concerns, leaving a heavy bill for the tax payers to pick up. Examples of such intervention are the rescue of the Government Sponsored Enterprises (GSE's) Freddie Mac and Fannie Mae and the insurance giant American International Group in the United States; the banks Northern Rock and Lloyds in the United Kingdom.

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The assertion that government intervention is unavoidable in the midst of a full blown financial crisis (since the costs of inaction can be considerably superior to the costs of action, due to contagion effects), moved the focus of regulation proposals towards *"measures to reduce and address the fiscal costs of future financial failures"* (*IMF*, 2010, p. 2). One of these measures is to involve the banks in the cost bearing of the financial system's rescue. This prompts the question of how to define this contribution. For the purpose of this paper we are mainly interested in the following topic[1]:

*"**Rate of the levy:** A uniform rate has the benefit of ease of implementation, but it does not contribute to reducing riskiness and systemicness. A risk-adjusted rate could be designed to address the contribution to systemic risk. Ideally, the rate would vary according to the size of the systemic risk externality, e.g., based on a network model which would take into account all possible channels of contagion." (IMF, 2010, p. 12).*

The network structure of the interbank market is a crucial point to contagion since it provides information on how the failure of a bank can spillover to its neighbours, potentially leading to the widespread collapse of the system. Imagine that bank A is connected to bank B that goes bankrupt and suppose also that this bankruptcy brings with it the bankruptcy of A. Now, suppose that C is not connected neither to A nor to B, then the impact on C is considerably different comparing to the case where a connection exists with A and/or B. The reasoning can be generalise to other banks that are in the same situation of C, fundamentally the network of connections is important to study how contagion can spread once an initial event triggers it.

The aim of this paper is to study the impact that the levy, 'network tax', has on the endogenous formation of the network. The tax is an exogenous shock to the environment that forces the system to adjust to it, changing its original configuration. In order to do so, we build on the model of *Castiglionesi and Navarro* (2010). Here, poorly capitalised banks (peripheral) gamble with their customers deposits to obtain private benefits. When well capitalised banks (core) weigh establishing credit lines with less capitalised banks they do so based on a trade-off between liquidity coinsurance and exposure to credit risk. A contributory regime that charges banks according to their exposure to contagion risk changes the trade-off that motivates the formation of the network in the first place. Now, banks that contribute more to the risk of contagion are assigned a heavier tax regime, which has the potential to shift their risky activities towards safer grounds. On the other hand, safer banks are forced to internalise the costs of connecting to risky neighbours, which makes

[1]This document was brought to our attention through the specialised website www.financialnetworkanalysis.com maintained by *Kimmo Soramäki*.

the conditions under which a safer bank is willing to connect to a risky neighbour more demanding, potentially leading to a less connected network. Most importantly, these changes make the network structure dependent on the tax, which in turn affects total welfare. Therefore, any attempt to design a policy function that fails to take this into account may not be optimal and welfare reducing.

In this model, each bank is composed of depositors and shareholders/investors. Depositors have the right to withdraw a conditional promised return, whereas shareholders are entitled the residual value after the banks' portfolio is liquidated and depositors are repaid in the final period, i.e., depositors are senior creditors with respect to all other (even with respect to interbank claims). Shareholders (that also play the role of managers) define banks' investment decisions. There are three alternatives: (i) a safe asset; (ii) a risky asset that yields the same expected return of the safe asset if it succeeds and nothing if it fails, but entitles the shareholders to a private benefit and; (iii) liquidity used as a precautionary measure to buffer an idiosyncratic liquidity shock faced in the interim period. To say that an investment is safe has, in this context, a very particular meaning. An asset is taken to be risky if its return is uncertain even if it is refinanced in the interim period.

Banks can establish relationships among themselves, that take the form of conditional credit lines, in order to obtain liquidity coinsurance since whatever their investment decision might be it needs to be refinanced in the interim period. When deciding their 'neighbours', banks trade-off the benefit of coinsurance with the potential exposure to contagion if the neighbouring bank invests in the risky asset, i.e., it is gambling. Since investors have limited liability, the investment in the risky asset occurs when the bank is poorly capitalised which is denoted as gambling.

The base model assumes that the credit lines are established before the shock is realised. However, there would be no great changes if the links were established after the banks realise their liquidity needs since the basic trade-off between coinsurance and counterparty risk remains largely unchanged. Another matter that deserves clarification is taking the network structure as given after banks realise their liquidity shocks and not doing the same after the introduction of the tax. The assumption here is that liquidity shocks are not anticipatable unlike the establishment of the levy. While banks have time to adjust to a new regulatory environment, the same cannot be said about liquidity shocks since the institutions will first try to obtain the liquidity from existing credit lines before establishing new ones. Again this is an assumption from the base model that can be relaxed at no great cost for our analysis since the change in the basic trade-off that affects the network after the introduction of the tax remains largely unaltered.

The literature on financial networks 'exploded' in the aftermath of the recent financial crisis. Most of the papers written on it analyse contagion effects (for a survey see *Allen and Babus* (2009)) and only few of them take into account the endogenous formation of the network (i.e., *Babus* (2006), *Leitner* (2005) and *Castiglionesi and Navarro* (2010)). The importance of endogenous network formation finds some coherence with the Lucas' critique *Lucas* (1976), i.e., when designing a policy its makers should take into account that if the system has time to adjust it will do just that. Although the idea that the network will respond endogenously to an exogenous shock is hardly new (see, for example, *Haldane* (2009) or *Allen and Babus* (2009)), to the best of our knowledge, this paper is the first to model specifically the changes in the network structure derived from the tax and the role that it plays in the definition of the policy function. The paper closest to ours is *Bluhm and Krahnen* (2010), the authors use a numerical model, that takes into account the interconnections among banks (interbank claims and portfolio selection decisions), to determine the optimal systemic risk charge (which we call 'network tax'). Although they also explain the intuition of how banks can adjust to the tax, they take the network structure as given which differentiates their work from ours. Furthermore, *Markose et al.* (2010, p. 16) also point out the potential effects that a similar tax would have in the Credit Default Swaps (CDS) market, but leave this issue for future research "Further experimentation is needed to consider the design of a reserve fund financed by a tax on 'super-spreaders' based on their 'Systemic Risk Ratio' and centrality statistics to mitigate the moral hazard problem currently being borne by the tax payer" The importance of modelling this issue as a network formation game is to inform policymakers on how their decisions will actually affect the behaviour they are trying to regulate, which can be a crucial factor in their success.

2 The Base Model

The model proposed by *Castiglionesi and Navarro* (2010) suggests as a motivation for banks to form links with each other the benefit of liquidity coinsurance. However, as undercapitalised banks' shareholders are assumed to derive a private benefit from gambling, a trade-off emerges in the network formation game. On the one hand, banks have the benefit of diversification of their liquidity shocks when establishing links with other banks. On the other hand, forming a link exposes a bank to the risk of a gambling neighbour. The setup we are using in this paper ignores the issue of capital transfers which is heavily studied in the base model.

2.1 Basic Setup

The events play out in five dates $t = 0,1,2,3,4$, where a continuum of consumers endowed with a monetary unit at $t = 0$ lie in a region contained in the set $N = \{1,2,...,n\}$. In each region there is a representative bank, where the consumers deposit their early endowment and withdraw only in the final date to consume. The bank i is also endowed in the initial period with capital, denoted by $e_i \in [0, \bar{e}]$ with $e = \{e_1, e_2, ..., e_n\}$, owned by its shareholders. The economy is thus represented by the pair (N, e). After the endowments are realised and the financial network is chosen (at $t = 0$), banks choose the projects that they invest in (at $t = 1$), offer deposit contracts to consumers (at $t = 2$), receive an idiosyncratic liquidity shock where liquidity is needed to refinance the investment that would otherwise fail (at $t = 3$) and finally returns materialise and agents consume (at $t = 4$). Banks have two illiquid investment opportunities: (i) a 'safe' project, denoted by b, that yields an expected return $\overline{R} > 1$ if it is refinanced in the advent of the liquidity shock and; (ii) a 'gambling' project, denoted by g, that yields $\overline{R}$ with probability ξ and 0 with probability $(1 - \xi)$ plus a certain private benefit bestowed on the shareholders denoted by $B > 0$. The distinguish characteristic between project b and g is that even if g is refinanced it can have a null return with positive probability.

The financial network, i.e., the set of sets of neighbouring banks are denoted by $K = \bigcup_{i \in N} K_i$ with $K_i \subseteq N$ and $k_i = \# K_i$. The authors only consider undirected networks, i. e., credit lines are reciprocal conditional on the realisation of the shock.

2.2 Network Structure and Liquidity Coinsurance

The liquidity shock determines the amount needed to refinance the illiquid asset, if the bank cannot refinance it then the return is lost. Therefore, in order to avoid losing the return, banks wish to establish connections (e.g. credit lines) with other banks that may receive a negatively correlated shock. Let $\varphi(k_i) = 1 - 2^{-k_i}$ denote the probability of a bank getting coinsurance with $\varphi'(k_i) > 0$ and $\varphi''(k_i) < 0$, with $f(k_i) = 1 + \varphi(k_i)$ denoting effect of coinsurance in the expected payoff.

Assuming perfect competition, depositors receive the full advantage from liquidity insurance, such that the depositors' expected payoff in bank i is given by: $D_i = [1 + \varphi(k_i)] R$, with R being the expected autarky return. Since investment in liquidity precludes the bank from achieving the maximum profitability from the safe asset we have $R < \bar{R}$. The proof of this can be found in *Castiglionesi and Navarro* (2010, p. 40), where the authors find that $R = \frac{1}{2}[(1 - \gamma)\bar{R} + \omega_H]$ with γ and ω_H denoting the optimal choice of liquidity and the high liquidity shock, respectively.

2.3 Network Structure and Counterparty Risk

At $t = 1$, banks make their investment decisions, let $s_i \in \{b, g\}$ be the project chosen by bank i. Taking the network and the strategy profile $s = \{s_i\}_{i \in N}$ as given, let $p_i(K, s)$ denote the probability that bank i does not go bankrupt, i.e., is able of fulfilling the amount contractualised to its depositors. Assuming:

$$p_i(K,s) = \begin{cases} \prod_{j\in K_i} \pi_j(s_j) & if \quad s_i = b \\ \xi \prod_{j\in K_i} \pi_j(s_j) & if \quad s_i = g \end{cases}, \tag{1}$$

with $\pi_j(s_j) = \begin{cases} 1 & if \quad s_j = b \\ \eta & if \quad s_j = g \end{cases}$.

The formulation chosen for p_i reflects that the bankruptcy probability is higher for a bank if it gambles, *ceteris paribus*. Furthermore, η can be interpreted as the risk of a gambling counterparty.

2.4 Expected Payoffs and Investment Project Choice

Since there are two types of agents in the economy, the expected payoff must be determined for shareholders and depositors. As it was assumed that the depositors receive the total gain of coinsurance[2], then their payoff is given by: $M_i(K,s) = p_i(K,s)D_i = p_i(K,s)f(k_i)R$. Another assumption is that $\xi R \geq 1$, this implies that even if the gambling bank is in autarky consumers will accept the deposit contract.

On the other hand, the equity stake has the following expected payoff:

$$m_i(K,e_i,s) = \begin{cases} p_i(K,s)\left[(1+e_i)f(k_i)R - D_i\right], & if \quad s_i = b \\ p_i(K,s)\left[(1+e_i)f(k_i)R - D_i\right] + B & if \quad s_i = g \end{cases}.$$

Denoting by g_i the number of gambling neighbours of bank i, we can rewrite equation (1) as follows: $p_i(K,s) = \begin{cases} \eta^{g_i} & if \quad s_i = b, \\ \xi \eta^{g_i} & if \quad s_i = g \end{cases}$

The decision to invest in the safe asset or to 'gamble' is crucially determined by the equity that investors/shareholders hold. Since their liability is limited, then they will gamble when the bank is undercapitalised. As in *Castiglionesi and Navarro* (2010, p. 11), bank will invest in the safe asset if and only if $\eta^{g_i} f(k_i)Re_i \geq \xi \eta^{g_i} f(k_i)Re_i + B \Leftrightarrow$

[2]If this assumption were to be relaxed one would expect a shift towards a safer system since the increase in shareholders' payoff would make the gambling project less attractive.

$$e_i \geq \frac{B}{(1-\xi)\, f(k_i)\, R\eta^{g_i}} = I^{\star}(k_i, g_i, \xi, \eta) \tag{2}$$

Next, the authors define the concept of *Investment Nash Equilibrium (INE) :*

Castiglionesi and Navarro (2010, p. 11) "An allocation (K,e,s) is an INE for a given economy (N,e), with $e=(e_i)_{i\in N}$, if

$$m_i(K,e_i,s) \geq m_i(K,e_i,(s_{-i},\widetilde{s}_i)) \qquad \forall\, i \in N,$$

with $\widetilde{s}_i \in \{b,g\}$. In other words, an allocation is an INE for a given economy if taking the financial network and capital as given there are no unilateral profitable deviations in the choice of the investment project. Note that an allocation (K,e,s) is an INE for a given economy if and only if $\forall \quad i \in N$

$$s_i = \begin{cases} b, & if \quad e_i \geq I^{\star}(k_i, g_i, \xi, \eta) \\ g, & if \quad e_i < I^{\star}(k_i, g_i, \xi, \eta) \end{cases},,$$

2.5 Network Formation Game - Decentralised Networks without Transfers

Following *Jackson and Wolinsky* (1996), the authors define the equilibrium concept.

Definition 1 (Definition 2 of *Castiglionesi and Navarro* (2010, p. 24)) *"An allocation without transfers (K,e,s) is pairwise stable (PSWT) if the following holds:*

1. *For all i and j directly connected in K: $m_i(K,e,s) \geq m_i(K\setminus ij, e, \tilde{s})$ and $m_j(K,e,s) \geq m_j(K\setminus ij, e, \tilde{s})$ for all allocations $(K\setminus ij, e, \tilde{s})$ that are INE;*

2. *For all i and j not directly connected in K: if there is an INE $(K\cup ij, e, \tilde{s})$ such that $m_i(K,e,s) <$ $m_i(K\cup ij, e, \tilde{s})$, then $m_j(K,e,s) > m_j(K\cup ij, e, \tilde{s})$."*

The first condition states that nodes can deviate unilaterally if the addition of the link is not beneficial to them. The second, expresses the idea that if there is a link that has not been established that is beneficial to one of the nodes, then it must be that the link would reduce the surplus of the other node.

The authors also introduce the definition of decentralised equilibrium, where

Definition 2 (Definition 3 of *Castiglionesi and Navarro* (2010, p. 24)) *"An allocation without transfers (K,e,s) is a decentralised equilibrium (DEWT) if it is an INE and PSWT".*

The financial network that emerges according to their prediction is one characterised by a core-periphery structure, as summarised in the following proposition

Proposition 1 (Proposition 7 of Castiglionesi and Navarro (2010, p. 24)**)** *Assume (N,e) define an economy without transfers. Then, a DEWT is a core-periphery structure, i.e., if (K^e,e,s^e) is a DEWT, then, for every pair of banks i and j such that $s_i^e = s_j^e = b$, we have that $i \in K_j^e$ and $j \in K_i^e$.*

The proof can be found in *Castiglionesi and Navarro* (2010, p. 56).

2.6 Four Banks Example

As an example, let us study the four banks case similar to the one presented in *Allen and Gale* (2000). Unlike the models inspired by *Diamond and Dybvig* (1983), here the question of bank runs does not pose itself. A bank only goes bankrupt in the last period, if it cannot pay the promised amount to its depositors. A bank can only go bankrupt if itself invests in the gambling project and the expected returns do not materialise or if it is linked to a neighbour bank where that happens.

In the interim period, banks need to refinance their investments. The precise amount is unknown in the initial period, so the funding needs are an idiosyncratic liquidity shock (ω) that can take two values: ω_H and ω_L, with $\omega_H > \omega_L$. When deciding which alternative to chose, banks also allocate some of their resources to liquidity, in order to self-insure partially against the idiosyncratic shock. However, as *Castiglionesi and Navarro* (2010, pp. 39-40) demonstrate, full self-insurance is not optimal if the liquidity shock is high enough.

Let us denote by $\gamma = (\omega_H+\omega_L)/2$ the optimal amount of self-insurance. Assuming that banks offer contingent deposits, i.e., the amount promised to depositors depends on the assets being able to refinanced or not. Therefore, the only possibility of a bank *i* being unable to repay its customers is if itself is liquidity endowed (it receives a low liquidity shock) and if it lends the surplus to a gambling counterpart that receives a high liquidity shock. If the counterpart's return came to be lost, then it is no longer able of repaying bank *i*.

If this were to happen, bank *i* has just the return of the initial investment (i.e., deposits plus capital minus investment in liquidity) in the illiquid asset $(1+e_i-\gamma)\bar{R}$ to repay its depositors, since the initial investment in liquidity (γ) is dispersed in the refinancing of the illiquid asset and in the interbank loan. It will go bankrupt if this amount falls short of the promised one (i.e., return of the part of deposits invested in the illiquid project plus the remaining invested in liquidity) that is $(1-\gamma)\bar{R}+\gamma-\omega_L$, i.e., $(1+e_i-\gamma)\bar{R}<(1-\gamma)\bar{R}+\gamma-\omega_L$.

which implies,

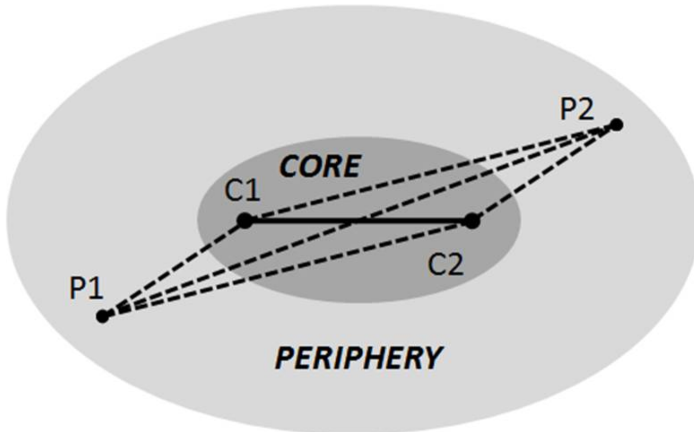
$$\overline{e_i} < \frac{\omega_H - \omega_L}{2\bar{R}}. \tag{3}$$

The expression for the limit capital for a bank that sees its assets' returns materialised and the loss of its interbank claims, has the underlying assumption that these claims are not diversified. Obviously, if a lender has its claims diversified the probability of being 'infected' by a troubled neighbour would decrease.

In terms of contagion, *Castiglionesi and Navarro* (2010) only take into account direct effects, i.e., contagion only occurs if a neighbour bank invests in the gambling project and fails to meet its obligations. The effect of the default of a neighbour's neighbour is excluded from the analysis, so systemic effects are ruled out. I.e., here, core banks are taken to be perfectly sound counterparties, but they can also go bankrupt if they are exposed to a gambling counterparty. However, due to the limited set of actions available to banks in the model (banks can either lend or borrow, not both), the failure of a core bank would not trigger any systemic effects since no other bank holds an interbank claim against it.

Furthermore, indirect linkages *via* asset prices are also excluded from the analysis. If banks tend to herd in terms of their investment decisions they are exposed to sudden price drops that are motivated by the need of troubled banks to raise liquidity quickly. That leads to a downward asset price spiral, that affects sound banks through mark-to-market accounting practises (see for example, *Allen and Carletti* (2008), *ECB* (2010) or *Cifuentes et al.* (2005)). This topic is left for future research.

Figure 1: Four Banks Example



Let us assume a network where there are two core banks and two peripheral ones that are all connected to each other (i.e., the network is complete, as depicted in figure (1)) and then define under what conditions this network endogenously emerges. Note that both core banks are connected to both peripheral banks in order to diversify their claims, i.e., it is more beneficial for a core bank to create a credit line with half of the value with two periphery banks than concentrate the same amount of potential credit in a single gambling counterpart, but it would rather be connected to another core bank. After doing so, let us study how resilient is the network to contagion. Denote by C_i with $i = 1,2$ the core banks and by P_j with $j = 1,2$ the peripheral ones.

Table 1: Idiosyncratic Liquidity Shocks

State	**Prob.**	P_1	P_2	C_1	C_2	**State**	**Prob.**	P_1	P_2	C_1	C_2
Ω_1	$1/16$	ω_L	ω_L	ω_L	ω_L	Ω_9	$1/16$	ω_H	ω_L	ω_H	ω_L
Ω_2	$1/16$	ω_H	ω_H	ω_H	ω_H	Ω_{10}	$1/16$	ω_H	ω_L	ω_L	ω_L
Ω_3	$1/16$	ω_L	ω_H	ω_H	ω_H	Ω_{11}	$1/16$	ω_H	ω_H	ω_L	ω_H
Ω_4	$1/16$	ω_H	ω_L	ω_H	ω_H	Ω_{12}	$1/16$	ω_H	ω_L	ω_L	ω_H
Ω_5	$1/16$	ω_L	ω_L	ω_H	ω_H	Ω_{13}	$1/16$	ω_L	ω_H	ω_H	ω_L
Ω_6	$1/16$	ω_L	ω_L	ω_L	ω_H	Ω_{14}	$1/16$	ω_L	ω_H	ω_L	ω_L
Ω_7	$1/16$	ω_L	ω_L	ω_H	ω_L	Ω_{15}	$1/16$	ω_H	ω_H	ω_H	ω_L
Ω_8	$1/16$	ω_L	ω_H	ω_L	ω_H	Ω_{16}	$1/16$	ω_H	ω_H	ω_L	ω_L

In order to do so, the $f(k_i)$ will be computed based on a complete mapping of the states of the world represented in table (1). Table (1) is composed of several blocks. The first one $[\Omega_1 - \Omega_2]$ corresponds to the states of nature where the links (i.e., credit lines) are not used since the liquidity shock is perfectly correlated across the system. The second $[\Omega_3 - \Omega_{15}]$ is composed by the states where peripheral banks are both borrowers and lenders, since there is a diversification of the interbank claims, the probability of contagion is lower. Finally, the block $[\Omega_{16}]$ is the case where gambling banks are just borrowers and core banks do not lend to each other. The final case is where one would expect contagion to emerge, which will be studied next.

Let us start by analysing the amount of liquidity exchanged in each state of the world. When a node receives a favourable liquidity shock, its endowment is $\gamma - \omega_L$ to distribute among its needy neighbours. We assume that banks wish to diversify their interbank claims, such that the promised amount allocated to each individual credit line never reaches this endowment. In their turn, banks negatively affected by the liquidity shock require $\omega_H - \gamma$ (which is the same as $\gamma - \omega_L$) to refinance their investment, otherwise the return is lost. For the sake of simplicity, let us assume contingent credit lines, i.e., if a bank has three links but only two need liquidity then the liquidity endowed bank lends the total amount to the two needy neighbours with the amount equally divided. Also, if a bank has three links and all of them require liquidity, then no coinsurance is possible. Basically, in this more restrictive notion of coinsurance, a bank gets it if it has two counterparties with a negatively correlated shock granted that there is sufficient liquidity available to it. The $\varphi(k_i)$ function thus become dependent on the network structure.

This approach is considerably different from the one used in *Castiglionesi and Navarro* (2010) in the calculation of the probability of coinsurance, since the authors do not take into consideration the effect of diversification. As a consequence, a property of $\varphi(k_i)$ changes slightly, now we have $\varphi'(k_i) \geq 0$. This has no effect on the consistency of the results.

A core bank with three links (two of which are with peripheral banks) can get coinsurance with probability of $1/4$ corresponding to the states $(\Omega_5, \Omega_7, \Omega_9, \Omega_{13})$, therefore $f_{C_i}(k_i = 3) = 5/4$ with $i = 1, 2$. Knowing this, we can find the equity threshold that motivates the banks to belong to the core, which is given by $e_i \geq \frac{4B}{5(1-\xi)R\eta^2}$. Similarly, one can find the benefits of coinsurance when the core bank is only connected to the other core bank and to a single gambling neighbour, i.e., $f_{C_i}(k_i = 2) = 9/8$ corresponding to the states (Ω_7, Ω_9) or alternatively to the states (Ω_7, Ω_{13}).

We now are interested in finding for what interval of values of η a core bank is interested in being connected to all other banks, and peripheral banks wish to be connected among themselves. A core bank wishes to connect to another gambling bank if the benefits from coinsurance outweigh the exposure to counterparty risk, i.e., if $\eta^{(g_i+1)} e_i f_C(k_i+1) R \geq \eta^{g_i} e_i f_C(k_i) R$.

which implies, $\eta \geq \frac{f(k_i)}{f(k_i+1)}$. More precisely, in our example, $\eta \geq \frac{9/8}{5/4} \Leftrightarrow \eta \geq 9/10$.

We turn now to the connection between peripheral banks. Two peripheral banks wish to be connected with each other if $\xi \eta^{(g_i+1)} e_i f_P(k_i+1) R + B \geq \xi \eta^{g_i} e_i f_P(k_i) R + B$.

which implies, $\eta \geq \frac{f_P(k_i)}{f_P(k_{i+1})}$. More precisely, in our example[3], $\eta \geq \frac{9/8}{5/4} \Leftrightarrow \eta \geq 9/10$. Gathering the two conditions we find that the network assumed is obtained if $e_i \geq \frac{4B}{5(1-\xi)R\eta^2}$ for the core banks (and otherwise for the two peripheral ones) and if $\eta \geq 9/10$.

Table 2: Maximum Loss Given Default

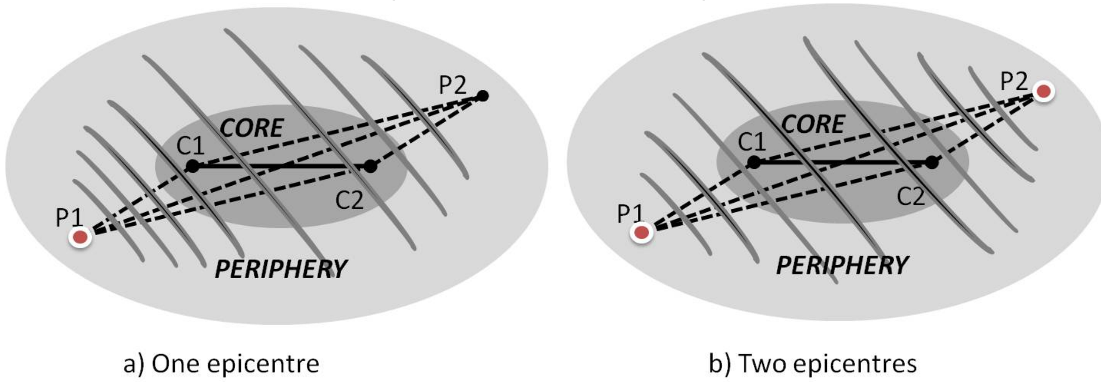
State	P_1	P_2	C_1	C_2
Ω_1	0	0	0	0
Ω_2	0	0	0	0
Ω_3	0	0	0	0
Ω_4	0	0	0	0
Ω_{11}	0	0	0	0
Ω_{15}	0	0	0	0
Ω_6	0	0	0	0
Ω_7	0	0	0	0
Ω_5	0	0	0	0
Ω_{10}	0	$\frac{\gamma-\omega_L}{3}$	$\frac{\gamma-\omega_L}{3}$	$\frac{\gamma-\omega_L}{3}$
Ω_{14}	$\frac{\gamma-\omega_L}{3}$	0	$\frac{\gamma-\omega_L}{3}$	$\frac{\gamma-\omega_L}{3}$
Ω_8	$\frac{\gamma-\omega_L}{2}$	0	$\frac{\gamma-\omega_L}{2}$	0
Ω_9	0	$\frac{\gamma-\omega_L}{2}$	0	$\frac{\gamma-\omega_L}{2}$
Ω_{12}	0	$\frac{\gamma-\omega_L}{2}$	$\frac{\gamma-\omega_L}{2}$	0
Ω_{13}	$\frac{\gamma-\omega_L}{2}$	0	0	$\frac{\gamma-\omega_L}{2}$
Ω_{16}	0	0	$\gamma-\omega_L$	$\gamma-\omega_L$

To study contagion, one must start by analysing the Loss Given Default (LGD) that emanates from the default event. The maximum LGD that a lender can expect at any state of nature is described in table (2), maintaining the assumption that there are no systemic effects 'infecting' core banks.

Table (2) is composed of several blocks, according to the LGD involved in those states of nature. A lender may have $LGD = 0$ in two circumstances, whether it has not lent any amount at all or it lent any amount to a core bank. The states $\{\Omega_{10}, \Omega_{14}\}$ are those where there are three lenders and only one borrower, the assumption is that each creditor lends the same amount to the borrower. In the states of nature $\{\Omega_8, \Omega_9, \Omega_{12}, \Omega_{13}\}$ there are two lenders and two borrowers, the assumption is the same as in the previous case. Ω_{16} is the state where the entire system has higher chances of collapsing since each lender can loose its entire interbank claims.

[3] The calculations of $f_P(k_i)$ are straightforward following the examples given for $f_C(k_i)$.

Figure 2: 'Wave' of Contagion



What is left now to do is to find with what probability contagion hits the system and how severe are the consequences. The epicentre of contagion can only be located in the periphery. Therefore, we can have a 'wave' of contagion with two potential epicentres, as depicted in figure (2).

Table 3: Capital Threshold

State	P_1	P_2	C_1	C_2
Ω_{10}	0	$\frac{\omega_H-\omega_L}{6\bar{R}}$	$\frac{\omega_H-\omega_L}{6\bar{R}}$	$\frac{\omega_H-\omega_L}{6\bar{R}}$
Ω_{14}	$\frac{\omega_H-\omega_L}{6\bar{R}}$	0	$\frac{\omega_H-\omega_L}{6\bar{R}}$	$\frac{\omega_H-\omega_L}{6\bar{R}}$
Ω_8	$\frac{\omega_H-\omega_L}{4\bar{R}}$	0	$\frac{\omega_H-\omega_L}{4\bar{R}}$	0
Ω_9	0	$\frac{\omega_H-\omega_L}{4\bar{R}}$	0	$\frac{\omega_H-\omega_L}{4\bar{R}}$
Ω_{12}	0	$\frac{\omega_H-\omega_L}{4\bar{R}}$	$\frac{\omega_H-\omega_L}{4\bar{R}}$	0
Ω_{13}	$\frac{\omega_H-\omega_L}{4\bar{R}}$	0	0	$\frac{\omega_H-\omega_L}{4\bar{R}}$
Ω_{16}	0	0	$\frac{\omega_H-\omega_L}{2\bar{R}}$	$\frac{\omega_H-\omega_L}{2\bar{R}}$

Following the reasoning similar to the used in subsection 2.6, we can calculate the threshold capital that separates a node from bankruptcy in the event of defaulting counterpart(s). Take for example state Ω_{10}, the bank C_1 has $(1+e_i-\gamma)\bar{R}+\gamma-\omega_L-\frac{\gamma-\omega_L}{3} \Leftrightarrow (1+e_i-\gamma)\bar{R}+\frac{2\gamma-2\omega_L}{3}$ available to repay its depositors, so it will go bankrupt if this amount falls short of the promised one that is $(1-\gamma)\bar{R}+\gamma-\omega_L$, i.e., $(1+e_i-\gamma)\bar{R}+\frac{\omega_H-\omega_L}{3} < (1-\gamma)\bar{R}+\gamma-\omega_L$. Which implies, $\bar{e}_i < \frac{\omega_H-\omega_L}{6\bar{R}}$. Results are summarised in table (3).

Let us focus on periphery-core contagion. The contagion mechanism is very simple. First, a peripheral neighbour fails with probability $(1-\xi)$ due to the choice of the gambling project. Then, this initial bankruptcy triggers a loss in the core bank(s) which it is connected to (as shown in table(2)). If this loss exceeds the capital buffer, then the core bank itself will go bankrupt. Since the parameters are unknown, let us assume that a core node goes bankrupt whenever the LGD is greater or equal to $\gamma - \omega_L$ and then find for what parameter values this actually happens.

This situation takes place whenever the capital threshold is never reached. Since we know that a core only is one if $e_i \geq \frac{4B}{5(1-\xi)R\eta^2}$, bankruptcy will always occur when this amount falls short of the capital threshold given by $\bar{e}_i < \frac{\omega_H - \omega_L}{2\bar{R}}$, i.e., if $e_i \in \left[\frac{4B}{5(1-\xi)R\eta^2}, \frac{\omega_H-\omega_L}{2\bar{R}}\right[$ which holds as long as $B < \frac{5(\omega_H-\omega_L)(1-\xi)\eta^2 R}{8\bar{R}}$.

We can now calculate the probability of contagion. Let A denote the event of C_1 and C_2 going bankrupt and F_j the event that the peripheral bank j has a non performing gambling project. Then, $P(A) = P(\Omega_{16}) \times P(F_2) \times P(F_1) = {}^1\!/_{16}(1-\xi)^2$.

By definition, $P(A) = 1 - \eta^2 \Leftrightarrow \eta = \sqrt{1 - {}^1\!/_{16}(1-\xi)^2}$, which respects the condition showed by the authors $\eta > \xi$ for $\xi \in [0, 1[$.

It is exactly in the calculation of this probability that we find the importance of the network structure. Suppose that although both core banks have two gambling neighbours those are not the same ones, i. e., there is not an indirect link between C_1 and C_2. Then, even if both peripheral banks connected to C_1 go bankrupt that does not imply that C_2 also fails, unlike the case of the complete network.

3 'Network Tax'

The purpose of this paper is to study a very simple application of a 'network tax', i.e., a tax that is levied based on the banks' contribution to the risk of contagion. The motivation to this analysis comes from the meeting of the G-20 in April 2010 as already stated in the introduction.

Although the model presented in *Castiglionesi and Navarro* (2010) does not take into account systemic effects, it does provide a simple structure that allows the study of what changes are to be expected when the links established become subject to a levy and how that affects the risk of direct contagion. Given this limited definition of risk of contagion, the investment decision will be the fundamental factor in the delimitation of the base of the tax, i.e., two banks each investing in the safe asset should not be tributated since their connection does not imply any contagion risk.

To simplify the analysis, let us assume that capital is perfectly observable such that investment decisions are also perfectly foreseeable. This assumption is no doubt extremely strong and will be relaxed in future research.

3.1 Tax Formulation

The fundamental issue involved in the definition of the tax is who should pay it. Should the full burden of the levy be supported by the gambling borrower? Or should the lender also contribute?

The purpose of such a contribution is to provide governments with the means to recapitalise the financial system if dire times were to present themselves (again). Therefore, if the burden were to be bestowed only to a fraction of those who benefit from the rescue, a free-rider problem could emerge. Such a tax could be used to minimise the underestimation of risk when a lender is connecting itself to an institution that is implicitly guaranteed by government intervention (i.e., *too-something-to-fail*[4]). In this paper we will assume that whenever there is a gambling bank involved in the link the tax must be paid by both parties.

The formulation chosen is a very general one given by a function $\rho_i(x)$, where x denotes those links that involve contagion risk and therefore should be levied on bank i (i.e., if a core bank establishes a link with a peripheral bank then it has to pay the tax on that link, whereas a gambling bank will always pay the tax in every link it establishes since it is the source of contagion risk). The only restriction we impose is that the tax collected be increasing in the potential exposure to risk, i.e., $\rho_i'(x) > 0$.

3.2 Impact on the Payoffs

For the sake of simplicity, let us assume that the full weight of the tax is bestowed on the shareholders. Therefore, their payoffs are now given by:

$$m_i(K, e_i, s, \rho) = \begin{cases} p_i(K,s)\left[(1+e_i)f(k_i)R - D_i\right] - \rho_i(g_i)g_i, & if \quad s_i = b \\ p_i(K,s)\left[(1+e_i)f(k_i)R - D_i\right] + B - \rho_i(k_i)k_i & if \quad s_i = g \end{cases}.$$

[4]As a great variety of such implicit guarantees have been studied in the literature, including here only the classic too-big-to-fail issue would be a restricting view. Therefore, designating this issue by too-something-to-fail englobates other concepts as the ones found in *Acharya and Yorulmazer* (2007) and in *Markose et al.* (2010).

3.3 Impact on Banks' Investment Decisions

The tax will also have an impact on investment decisions. Since we have assumed that as gambling banks are *per se* a source of risk, the decision to gamble brings with it a more heavy tax regime than the one allowed by the alternative investment opportunity. Which can be confirmed by re-writing equation (2):

$$\eta^{g_i} f(k_i) Re_i^{tax} - \rho_i(g_i) g_i \geq \xi \eta^{g_i} f(k_i) Re_i^{tax} + B - \rho_i(k_i) k_i \Leftrightarrow$$

$$\eta^{g_i} f(k_i) Re_i^{tax} \geq \xi \eta^{g_i} f(k_i) Re_i^{tax} + B + \rho_i(g_i) g_i - \rho_i(k_i) k_i$$

which implies,

$$e_i^{tax} \geq \frac{B}{(1-\xi) f(k_i) R\eta^{g_i}} + \frac{\rho_i(g_i) g_i - \rho_i(k_i) k_i}{(1-\xi) f(k_i) R\eta^{g_i}} = I^\star(k_i, g_i, \xi, \eta, \rho) \tag{4}$$

Since some changes are introduced into equation (2), we define the concept of *Investment Nash Equilibrium after Levy (INEL) :*

An allocation (K, e, s, ρ) is an INEL for a given economy (N, e), with $e = (e_i)_{i \in N}$, if

$$m_i(K, e_i, s, \rho) \geq m_i(K, e_i, (s_{-i}, \widetilde{s}_i), \rho) \qquad \forall\, i \in N,$$

with $\widetilde{s}_i \in \{b, g\}$. In other words, an allocation is an INEL for a given economy if taking the financial network, capital and contributory regime as given there are no unilateral profitable deviations in the portfolio allocation. An allocation (K, e, s, ρ) is an INEL for a given economy if and only if $\forall \quad i \in N$

$$s_i = \begin{cases} b, & if \quad e_i \geq I^\star(k_i, g_i, \xi, \eta, \rho) \\ g, & if \quad e_i < I^\star(k_i, g_i, \xi, \eta, \rho) \end{cases}$$

Taking into account that $\rho'(x) > 0$ and $G_i \subseteq K_i$ such that $k_i \geq g_i$, then the minimum level of capital that motivates the investment in the safe asset with the tax (as given by equation (4)) is lower than previously (as given by equation (2)).

3.4 Impact on Network Formation

Since the 'network tax' changes the trade-off that motivates the formation of the network, it is here that the fundamental impact hits.

Before the introduction of the levy, the trade-off consisted in exchanging the benefit of the increase of the probability of coinsurance with the cost of exposure to the risk of failure induced by a gambling neighbour. Now, there is an added cost independent of the nodes' investment decision.

Definition 3 *An allocation without transfers after Levy* (K,e,s,ρ) *is pairwise stable (PSWTL) if the following holds:*

1. *For all i and j directly connected in K:* $m_i(K,e,s,\rho) \geq m_i(K \setminus ij, e, \tilde{s}, \rho)$ *and* $m_j(K,e,s,\rho) \geq m_j(K \setminus ij, e, \tilde{s}, \rho)$ *for all allocations* $(K \setminus ij, e, \tilde{s}, \rho)$ *that are INEL;*

2. *For all i and j not directly connected in K: if there is an INEL* $(K \cup ij, e, \tilde{s}, \rho)$ *such that* $m_i(K,e,s,\rho) < m_i(K \cup ij, e, \tilde{s}, \rho)$*, then* $m_j(K,e,s,\rho) > m_j(K \cup ij, e, \tilde{s}, \rho)$.

The explanation of both statements is similar to the one presented in Definition 1. The only difference lies on the fact that the trade-off is now altered with the introduction of the tax. Therefore, a new concept of equilibrium is needed:

Definition 4 *An allocation without transfers after Levy* (K,e,s,ρ) *is a decentralised equilibrium (DEWTL) if it is an INEL and PSWTL.*

3.4.1 Core-Core Relations

Let us start by analysing the interconnectedness of the core. Now, $p_i(\cdot) = 1$ and $m_i(\cdot) = e_i[1+\varphi(k_i)]R$. Since indirect contagion is ruled out, establishing a link with a core bank does not pose any risk of contagion. Therefore, the original argument establishing that banks in the core were all connected to each other relied on the fact that since they all were investing in the safe asset no marginal cost existed from creating a new link remains valid. Therefore, Proposition 1 remains unaltered.

3.4.2 Relations involving Peripheral Nodes

The effect of the tax is only visible on relations involving at least one gambling bank as that is the only source of risk of contagion in the model. The next proposition summarises the results:

Proposition 2 *The conditions for the establishment of links with peripheral nodes are more stringent with the introduction of the Levy.*

The proof follows.

The condition establishing the optimality of a link being established between a core bank and a periphery one is given by: $\eta^{(g_i+1)} e_i^{tax} f(k_i+1) R - (g_i+1)\rho_i(g_i+1) \geq \eta^{g_i} e_i^{tax} f(k_i) R - \rho_i(g_i) g_i$.

which implies,

$$\eta \geq \frac{f(k_i)}{f(k_i+1)} + \frac{(g_i+1)\rho_i(g_i+1) - \rho_i(g_i) g_i}{\eta^{g_i} e_i^{tax} f(k_i+1) R} \tag{5}$$

Comparing equation (5) with the one that would be obtained in the absence of the tax and assuming that $g_i = 0$, i.e., the maximum counterparty risk accepted by a core bank when deciding to connect to a single gambling neighbour is lower as the incorporation of the tax affects the trade-off that motivates the formation of the network.

The condition establishing the optimality of a link being established between a periphery bank and a core one is given by: $\xi \eta^{g_i} e_i^{tax} f(k_i+1) R + B - (k_i+1)\rho_i(k_i+1) \geq \xi \eta^{g_i} e_i^{tax} f(k_i) R + B - \rho_i(k_i) k_i$.

which implies,

$$\xi \eta^{g_i} e_i^{tax} R\left[f(k_i+1) - f(k_i)\right] \geq (k_i+1)\rho_i(k_i+1) - \rho_i(k_i) k_i \tag{6}$$

In its turn, equation (6) tells us that since now the gambling bank has to pay tax whenever it creates a link, even connecting to a core bank is a decision that involves a cost-benefit analysis.

In its turn, the condition establishing the optimality of a link being established between two periphery banks is given by: $\xi \eta^{(g_i+1)} e_i^{tax} f(k_i+1) R + B - (k_i+1)\rho_i(k_i+1) \geq \xi \eta^{g_i} e_i^{tax} f(k_i) R + B - \rho_i(k_i) k_i$.

which implies,

$$\eta \geq \frac{f(k_i)}{f(k_i+1)} + \frac{(k_i+1)\rho_i(k_i+1) - \rho_i(k_i) k_i}{\xi \eta^{g_i} e_i^{tax} f(k_i+1) R} \tag{7}$$

Comparing equation (7) with the one that would be obtained in the absence of the tax and assuming that $g_i = 0$, i.e., the maximum counterparty risk accepted by a periphery bank when deciding to connect to a single gambling neighbour is lower as the incorporation of the tax affects the trade-off that motivates the formation of the network. ■

3.5 Four Banks Example Revisited

The purpose of this subsection is to compare the revised example presented in subsection 2.6 before and after the tax is introduced.

The first step is to analyse under what conditions the complete network may arise. The tax motivates nodes to choose the safe project, because otherwise they will face a heavier tax regime. Therefore, the minimum amount of capital that separates core and peripheral banks is lower and given by: $e_i^{tax} \geq \frac{4B}{5(1-\xi)R\eta^2} + \frac{4[\rho_i(2)-\rho_i(3)]}{5(1-\xi)R\eta^2}$. Now, we turn our attention to the conditions under which banks wish to connect to each other. Since the tax does not affect the conditions that motivate core banks to connect to each other, all core banks are connected among themselves. The same is not true for links that involve at least one gambling bank, as it is summarised below:

- Core-periphery relations: $\eta \geq \frac{9/10+\sqrt{81/100+16\frac{[2\rho_i(2)-\rho_i(1)]}{5e_i^{tax}R}}}{2} > 9/10$.
- Periphery-core relations: $\eta \geq \sqrt{\frac{8[3\rho_i(3)-2\rho_i(2)]}{\xi e_i^{tax}R}}$.
- Periphery-periphery relations: $\eta \geq 9/10 + \frac{4[3\rho_i(3)-2\rho_i(2)]}{5\xi e_i^{tax}R} > 9/10$.

The tax makes the conditions that need to be met for the complete network to form more stringent, i. e., if the tax is large enough then the network can be completely safe and contagion happens with probability zero. However, this only happens if we have a empty periphery, which means depositors and shareholders have now less opportunities of coinsurance leading to potentially lower payoffs.

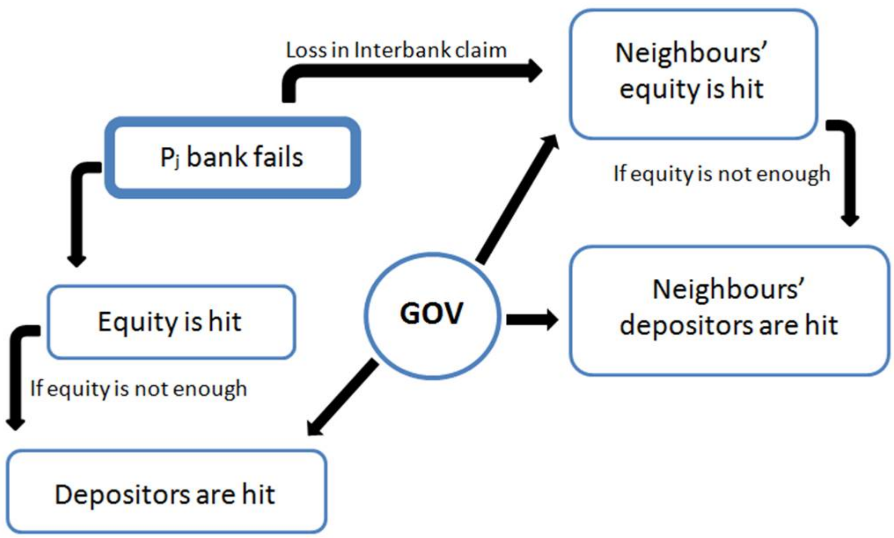
In equilibrium, there is a trade-off between risk of contagion and benefits of coinsurance. This is the basic idea that underlies the definition of a welfare function similar to the one presented in *Castiglionesi and Navarro* (2010, p. 11). Such a function must express the sum of of payoff earned by depositors and shareholders weighted by the probabilities of occurrence of each state of nature, as given by equation (8):

$$\mathbf{W} = \sum_{i=1}^{n} \left[\delta_i^d M_i(K,s) + \delta_i^{sh} m_i(K,e_i,s) + \delta_{gov} \sum_{i=1}^{n} \rho_i(base_i)\, base_i \right] \quad (8)$$

where δ_i^d, δ_i^{sh} and δ_{gov} denote the weights attributed by the policymaker to the surplus of the depositants, shareholders of bank i and to the government, respectively. $base_i$ denotes the number of links that are tributated in the case of bank i. The solvency fund here takes the form of a all purpose fund that is bestowed upon the government such that it can use it to recapitalise the system when needed.

Since there is a tax created to build a fund used to recapitalise banks and eventually repay depositors 'infected' by the original bankruptcy, in those states of nature where contagion arises the corresponding payoffs must be supplemented by government transfers that use the solvency fund's resources, as depicted in figure (3). This is expressed in the calculation of δ_{gov}.

Figure 3: Bankruptcy Mechanism with Solvency Fund



Note that the welfare function can only be evaluated after the network is formed, which is dependent on the choice of the tax formulation. The next subsection provides clues to selection process of the tax formulation.

3.6 Selection of the Tax Formulation

The main contribution of this paper is to show that the selection of the tax formulation is affected by the endogenous formation of the network, unlike in *Bluhm and Krahnen* (2010) where the network is taken to be stationary when the policy function is defined. To do so, we propose an iterative algorithm were the network is taken to be stationary only locally to the purpose of evaluating total welfare and then is iterated until a superior policy formulation is found that leads to a pairwise stable network.

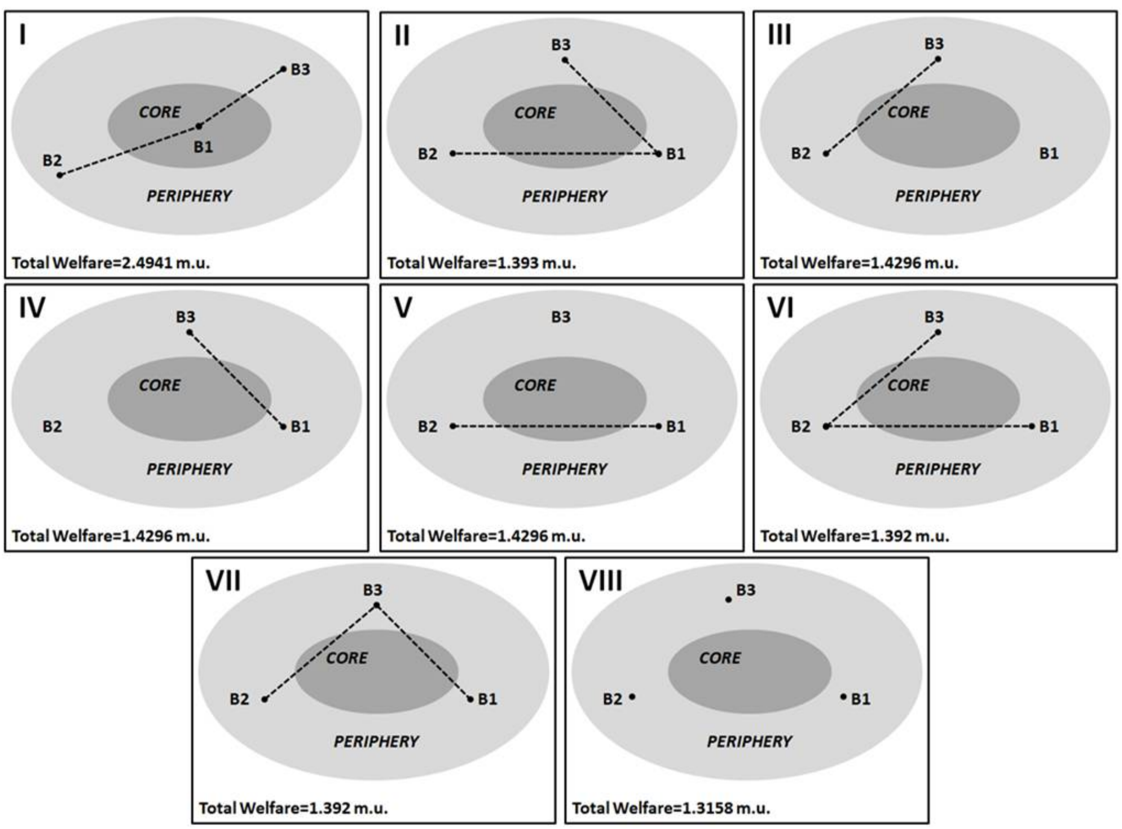
3.6.1 Steps

1. Hold capital fixed;
2. Guess $\rho(x)$, with $x = 1, ..., n-1$;
3. Check all networks that can be formed given $\rho(x)$, i. e., what banks lie in the core/periphery;
4. Check the Participation Constraints (Pairwise Stable networks);
5. Evaluate total welfare;
6. Go back to 2. until all proposed values are analysed.

3.6.2 Three Banks Example

To illustrate how the algorithm might work, we present a three banks example. The choice of three banks, and not four has been used until now, rests on the fact that graphical presentation of the results is not feasible in the latter case.

Figure 4: Feasible Networks



As it was done, the implementation of the algorithm is not very efficient. Basically, all possible networks were enumerated[5] and then restricted according to the constraints in steps 3 and 4. Once the set of feasible networks was defined, total welfare was evaluated for each policy function.

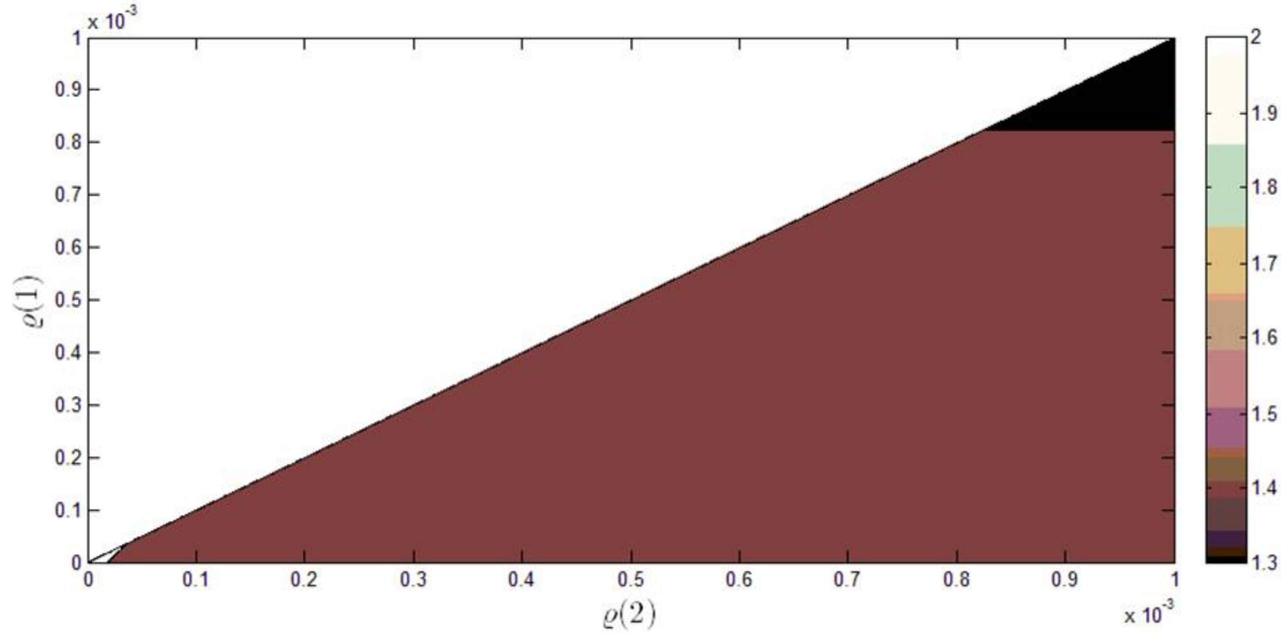
The parametrisation was chosen such that the network I depicted in figure (4) was feasible under a null tax, parameters are shown in table (4). Furthermore, no diversification of interbank claims was allowed in this example since there are only three banks. Finally, equal weights were assumed such that total welfare is just the sum of the individual welfare measures. The capital levels assumed for each bank were the following: $e_1 = 0.0255$ and $e_2 = e_3 = 0.0116$.

Table 4: Parametrisation

Parameter	Value
η	0.91
ξ	0.3124
R	1.3
$\bar{R}$	3.1429
B	0.0259
ω_H	0.4
ω_L	0.2
γ	0.3

[5]To do so, the methodology proposed in *Ruskey and Williams* (2009) was used. The translation into c code was conducted by Prof. Paulo S. A. Sousa (INESC-Porto, Portugal) for the project *Fique and Sousa* (2009) funded by FCT grant no. BII/UNI/4089/EEI/2008 and then translated by Fique, J.P. for the use of this paper.

Figure 5: Total Welfare as a Function of the Policy Function



Since there are multiple feasible networks that are pairwise stable, for the purpose of the determination of the policy function we ask what is the maximum total welfare that can be achieved given the contributory regime.

Figure 6: Total Number of Connections as a Function of the Policy Function

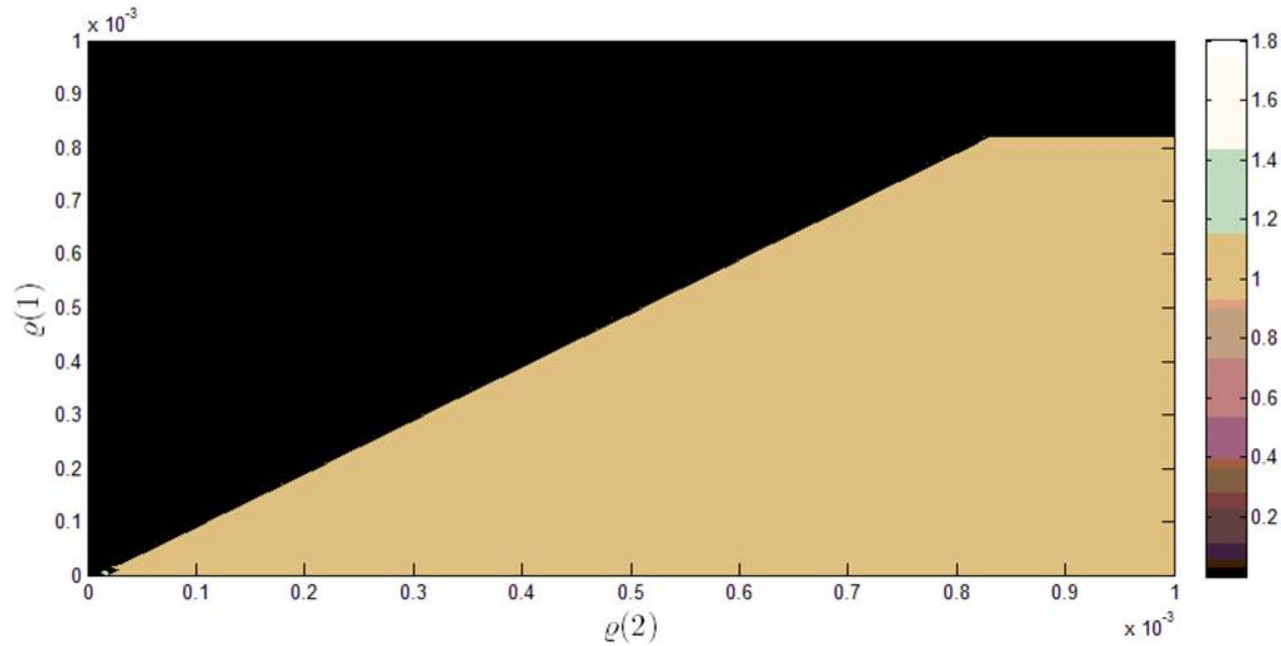


Figure (5) depicts total welfare as a function of the tax structure. Since we assume equal weights, then welfare remains unchanged for a given network, only its components change. We can see that there are three different levels of maximum welfare that can be achieved. If the tax is close to 0 then the network I depicted in figure (4) can still be maintained - with area. However, as the tax increases only networks (e.g., III, IV and V) with a single link between peripheral banks are feasible and respect the participation constraints - light brown area. Furthermore, if the tax becomes too high, then only the empty network where all autarky banks are peripheral can be sustained - heavy brown area. This can be confirmed by figure (6) that depicts the total number of connections for each policy function.

These results should not be interpreted as a defence of a close to zero tax, they only reflect the parametrisation chosen for the network and for the welfare function. The point we are trying to make is to show that a higher tax brings with it a less connected network and that the choice of the policy function must take into account how these effects affect the network. To do so, the understanding of how the network forms (or how it re-forms) is invaluable.

4 Conclusion

The main contribution of this paper is to show that a contributory regime based on exposure to contagion risk can affect the structure of the network and that these changes can play an important role in the design of the optimal policy function.

A secondary contribution is a methodological one. Exercises similar to the one presented in this paper may prove valuable in the choice of the methodological tools employed in the analysis, i. e., if we are concerned with the adjustment process triggered by an exogenous shock and not only how the new equilibrium network will look like, then we should favour dynamic models and equilibrium concepts more suited for that particular case.

The model presented in *Castiglionesi and Navarro* (2010) has several limitations though, as it only incorporates the effects of direct contagion and precludes systemic risk which is determinant to a more complete analysis of this 'network tax'. Also, links do not necessarily correspond to interbank exposures, they take the form of credit lines that may or may not be used conditional on the nodes connected receiving a negatively correlated liquidity shock, so our proposal to tax them is not entirely correct. However, the analysis carried out through this paper remains valid if we were talking about actual interbank claims, since our point is that the tax changes the trade-off between liquidity coinsurance and counterparty risk that motivates the formation of the network in the first place. Although the base model has some limitations, it does provide a simple structure that allows the study of what changes are to be expected when the links established become subject to a levy and how that affects the re-formation of the network after the contributory regime is established.

Future research would include the study of systemic risk in a dynamic environment where the transition process can be studied. Furthermore, our analysis suggests that asymmetric information would bring with it serious complications, further inquiry should also approach this issue.

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