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Panic and propagation in 1873: a computational network approach

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Abstract

We assess systematic risk in the U.S. banking system before and after the Panic of 1873 using a combination of linear programming and computational optimization to estimate the interbank network based upon total gross and net positions of national banks a week before the crisis. We impose various liquidity shocks resembling those of 1873, and find the network can capture the distribution of interbank deposits a year later. The network may be used to predict banks likely to panic (i.e., change reserve agent) in the crisis. The identified banks see their balance sheets weaken in the year after the crisis more than other banks. The results shed light on the nature and regional pattern of withdrawals that may have occurred in a classic 19th century U.S. financial crisis.

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1 Introduction

Understanding the susceptibility of financial systems to systemic risk, and particularly the contribution of interbank credit relationships to that risk, has taken on renewed urgency in the wake of the 2008 global financial crisis. With a few notable exceptions, however, empirically analyzing actual interbank relationships presents a challenge given that total gross and net positions of banks can typically be observed from balance sheets but detailed information about particular correspondents is usually confidential.¹ Moreover, the rise of complex financial assets and increases in off-balance-sheet activities make it nearly impossible to assess risk based on financial statements alone. This was less the case for the United States in the early 1870s when the vast majority of banks were organized under national charters. This "National Banking System" experienced a panic and crisis in 1873 that shares key features with 2008, including credit shortages and system-wide stress driven by speculation in assets that turned out to be overvalued. At the same time, the simpler portfolios held by U.S. banks in this period render their balance sheets, which survive from the early fall of each year as recorded in the Annual Reports of the Comptroller of the Currency, more informative of their condition than their modern-day counterparts. Less developed systems for the transfer of information and goods also made physical distance more important then for choosing interbank partners than it is now – while the internet and electronic market places now make it easy for banks to interact with any counterparty in the world, this was not the case in 1873.

We use these analytical advantages together with the regulatory constraints of the time to assess systemic risk in the U.S. banking system before, during, and after the Panic of 1873. We do this through a unique combination of linear programming and computational optimization, implemented on a high performance computer cluster, that allows us to estimate the interbank network and the parameters of a given utility function for banks simultaneously using balance sheet data reported a week before the start of the

¹See Mistrulli (2011) for an example using the types of more detailed information that is often restricted to regulators, e.g., Basel Committee on Banking Supervision (2015). There is also an active theoretical literature addressing the effects of financial contagion on networks, for example Acemoglu et al. (2015). See Glasserman and Young (2016) for a review of both aspects.

crisis.² Using this network, we show that, consistent with historical observation, direct counter-party risk was small, i.e., an unrealistically large shock would be required for a significant number of banks to fail. Indeed, losses from the few banks that failed during the crisis played a minor role compared to the liquidity shortages in New York and elsewhere caused by spontaneous deposit withdrawals, temporary suspensions of convertibility of bank notes into specie, and their eventual effects on the distribution of interbank deposits across banks. Comparisons of our network for 1873, after imposing various liquidity shocks, with actual post-crisis balance sheets from the fall of 1874 show that our network predicts which banks became subject to panics with striking accuracy. The results increase our understanding of a major historical crisis by demonstrating the robustness of the National Banking System, despite its apparent deficiencies, and point to the usefulness of network analysis in separating counterparty risk from other systemic components when investigating disturbances in modern banking systems.

Our approach is deliberately broad. Other recent studies (Calomiris and Carlson, 2016; Paddrick et al., 2016) using rich data on actual interbank balances from national bank examiners' reports stored in the U.S. National Archives offer insights on aspects of the network, but the archives are incomplete and the data insufficiently broad in geographic coverage or synchronized in timing to permit comprehensive analysis of a single nationwide event. Paddrick et al. (2016), for example, shed light on how interbank relationships and the associated balances changed between banks in New York and a set of correspondents in Philadelphia and Pittsburgh between 1862 and 1868, and in turn with those country banks that used these Pennsylvania banks as redeeming agents. Our approach, in contrast, while not using specific pairwise interbank positions, analyzes the entire banking *system* of the time. We do this using the structure of reserve requirements and the gross interbank positions of all national banks between the fall of 1873 and 1874 to build a system of interbank linkages that, given the 1873 shock, accurately lines up with observed changes in aggregate interbank balances.

 $^{^2\}mathrm{The}$ final results presented in this paper require approximately 130,000 hours of computation time to calculate.

The crisis of 1873 was among the more severe under the National Banking System, which was in effect from 1863 until the founding of the Federal Reserve in 1913. The National Banking Laws of 1863 and 1864 created the new system to finance the Union effort during the Civil War by requiring bank notes be secured with federal bonds rather than the varied collections of assets that individual state banking authorities had previously accepted as collateral. Federal control of the collateral required to issue bank notes was an improvement over the earlier system in which banks were granted charters by state legislatures and often based on political considerations, or under state free banking laws where weaker collateral was often accepted. The National Banking Laws also imposed a 10 percent tax on note issues of existing state-chartered banks, which led nearly all to either exit or convert to national charters by 1866, and created a Federal agency, the Office of the Comptroller of the Currency, to administer and provide oversight for the system.

The period leading up to the crisis can be characterized as one of general overbuilding in railroads financed by securities for which the risks were insufficiently understood. The resulting balance sheet vulnerabilities among banks in New York City, which had made large call money loans to support these projects, eventually led investors throughout the nation to re-evaluate their portfolios, and the failure of the investment bank Jay Cooke & Company on 18 September 1873 served as a catalyst for the crisis. Although Cooke was not a national bank but rather a private trust, its failure was followed by large withdrawals of individual and interbank deposits from several New York banks which, according to Sprague (1910, p. 15), had been "directly responsible for the satisfactory working of the credit machinery of the country."Interestingly, only 101 banks (1.68 percent) including seven national banks closed throughout the country in direct response to the panic (Wicker, 2000, pp. 6, 143), yet the consequences of the crisis on the distribution of interbank deposits throughout the nation turned out to be quite substantial.

2 Model

The analysis considers the network and distribution of interbank deposits among national banks before and after the crisis. In carrying out its monitoring role, the Comptroller required banks to submit reports of their condition at periodic "call dates," and these reports provide the raw data for our analysis. In this section we describe the data and how they are used to estimate the interbank network.

2.1 Data

The networks are based on balance sheets published in the Annual Report of the Comptroller of the Currency for the years 1873 and 1874.³ The 1873 report includes data for all 1,976 national banks operating in the United States on 12 September 1873, while the 1874 report includes the 2,001 operating national banks on 2 October 1874. These banks account for more than 88 percent of bank capital in the United States in 1874.⁴ The first date occurs shortly before the failure of Jay Cooke & Company (Sprague, 1910), and offers a benchmark for the condition of the banking system before the crisis.⁵ The 2 October 1874 date represents the nearest reporting date a year after the shock. The balance sheet data were collected using optical reading software and then checked by hand. Latitudes and longitudes of all banks were obtained using Google maps and measured at the geographic center of each municipality.

Each balance sheet includes information on a bank's total interbank deposits. The lia-

³https://fraser.stlouisfed.org/title/?id=56#!19089

⁴The rise of deposit banking and checking services made note issuance less important as a source of bank profits as the 19th century progressed, and eventually fueled a resurgence of State banks outside of the National Banking System after 1885. There had already been some gains by 1874, however, with the number of State banks reaching 551 (or nearly 22 percent of the total). These state banks are excluded from our network analysis due to lack of complete and detailed balance sheet data, but were generally much smaller with an average capital of \$108 thousand compared to nearly \$250 thousand for national banks (*Annual Report of the Comptroller of the Currency 1877*, pp. IV, XCI).

⁵Trusts were already an important part of the financial system in 1873, especially in New York, and it is not surprising that a trust company was at the center of the initial shock. Given that there were only 35 trusts nationwide, however, and all were located in the Northeast and with an average capital of \$625 thousand, they could not have been a key element in the redistribution of national bank deposits that we examine (Annual Report of the Comptroller of the Currency 1877, p. XCII).

bilities side shows the value of deposits "due to national banks," while the assets side lists deposits "due from other national banks"⁶ and "due from redeeming agents." Redeeming agents were themselves national banks located in designated "reserve cities," which played a specific role in the regulatory mechanism.⁷ Under the National Banking Laws, banks outside of reserve cities were required to keep 15 percent of their circulating notes and customer deposits as cash, two thirds of which (i.e., 9 percent) could be kept as deposits at national banks in the reserve cities. The reserve city banks were themselves required to keep 25 percent of their circulating notes and deposits as cash but half of this (i.e., 12.5 percent) could be kept in banks in the *central reserve city* - New York. National banks in New York City were then required to keep 25 percent of their circulating notes and deposits as cash. For non-reserve city (i.e., "country") banks, the "due from redeeming agents" item contains the total amount the bank had on deposit in national banks in reserve cities and the central reserve city banks it represented the amount deposited in New York City.⁸ These rules, as reflected in the reported interbank balances, provide the structure for the network model we describe below.

Table 1 summarizes the interbank positions of banks in each reserve city in 1873 and 1874. New York, as the central reserve city, is the largest holder of interbank deposits, with its total position more than four times that of Boston, the next largest city. The remaining reserve cities sizes are widely distributed from San Francisco with only two banks and \$220k of interbank deposits to Boston with 51 banks (more than New York) and \$16m of interbank liabilities. The significant impact of the panic on interbank deposits in New York City is clearly seen, with a reduction of approximately \$6m. This reduction was greater than observed across the banking system as a whole, meaning that a significant amount was redistributed with Boston and Chicago as the primary beneficiaries. Unfortunately

 $^{^6\}mathrm{The}$ 1874 report replaces this entry with "due from other banks and bankers " which additionally includes state banks.

⁷The reserve cities in both 1873 and 1874 were Albany, Baltimore, Boston, Chicago, Cincinnati, Cleveland, Detroit, Louisville, Milwaukee, New Orleans, Philadelphia, Pittsburgh, San Francisco, St. Louis, and Washington, D.C.

⁸This value is zero by definition for the national banks in New York City.

		Due from	Due from	Due to Other
1873	Banks	Redeeming Agents	Other Banks	National Banks
Albany	7	2,891	1,042	2,549
Baltimore	14	$2,\!629$	816	2,990
Boston	51	$8,\!905$	3,263	16,063
Chicago	19	3,748	$1,\!605$	$6,\!114$
Cincinnati	5	1,669	602	$3,\!178$
Cleveland	6	613	520	357
Detroit	3	526	432	322
Louisville	6	330	189	412
Milwaukee	4	732	152	756
New Orleans	9	798	585	465
New York	48	-	17,818	71,574
Philadelphia	29	$5,\!478$	3,818	7,024
Pittsburgh	16	2,265	814	1,546
San Francisco	2	413	192	22
St. Louis	8	1,293	321	1,791
Washington	3	161	33	79
Sum	230	32,450	32,199	115,240
				,
		Due from	Due from	Due to Other
1874	Banks	Due from Redeeming Agents	Due from Other Banks	Due to Other National Banks
1874 Albany	Banks 7	Due from Redeeming Agents 4,415	Due from Other Banks 2,999	Due to Other National Banks 2,321
1874 Albany Baltimore	Banks 7 14	Due from Redeeming Agents 4,415 2,166	Due from Other Banks 2,999 816	Due to Other National Banks 2,321 1,577
1874 Albany Baltimore Boston	Banks 7 14 58	Due from Redeeming Agents 4,415 2,166 9,770	Due from Other Banks 2,999 816 3,249	Due to Other National Banks 2,321 1,577 20,075
1874 Albany Baltimore Boston Chicago	Banks 7 14 58 19	Due from Redeeming Agents 4,415 2,166 9,770 4,907	Due from Other Banks 2,999 816 3,249 2,925	Due to Other National Banks 2,321 1,577 20,075 7,247
1874 Albany Baltimore Boston Chicago Cincinnati	Banks 7 14 58 19 5	Due from Redeeming Agents 4,415 2,166 9,770 4,907 1,539	Due from Other Banks 2,999 816 3,249 2,925 783	Due to Other National Banks 2,321 1,577 20,075 7,247 2,546
1874 Albany Baltimore Boston Chicago Cincinnati Cleveland	Banks 7 14 58 19 5 6	Due from Redeeming Agents 4,415 2,166 9,770 4,907 1,539 903	Due from Other Banks 2,999 816 3,249 2,925 783 466	Due to Other National Banks 2,321 1,577 20,075 7,247 2,546 265
1874 Albany Baltimore Boston Chicago Cincinnati Cleveland Detroit	Banks 7 14 58 19 5 6 3	Due from Redeeming Agents 4,415 2,166 9,770 4,907 1,539 903 743	Due from Other Banks 2,999 816 3,249 2,925 783 466 626	Due to Other National Banks 2,321 1,577 20,075 7,247 2,546 265 452
1874 Albany Baltimore Boston Chicago Cincinnati Cleveland Detroit Louisville	Banks 7 14 58 19 5 6 3 9	Due from Redeeming Agents 4,415 2,166 9,770 4,907 1,539 903 743 407	Due from Other Banks 2,999 816 3,249 2,925 783 466 626 707	Due to Other National Banks 2,321 1,577 20,075 7,247 2,546 265 452 896
1874 Albany Baltimore Boston Chicago Cincinnati Cleveland Detroit Louisville Milwaukee	Banks 7 14 58 19 5 6 3 9 4	Due from Redeeming Agents 4,415 2,166 9,770 4,907 1,539 903 743 407 463	Due from Other Banks 2,999 816 3,249 2,925 783 466 626 707 234	Due to Other National Banks 2,321 1,577 20,075 7,247 2,546 265 452 896 395
1874 Albany Baltimore Boston Chicago Cincinnati Cleveland Detroit Louisville Milwaukee New Orleans	Banks 7 14 58 19 5 6 3 9 4 7	Due from Redeeming Agents 4,415 2,166 9,770 4,907 1,539 903 743 407 463 209	Due from Other Banks 2,999 816 3,249 2,925 783 466 626 707 234 285	Due to Other National Banks 2,321 1,577 20,075 7,247 2,546 265 452 896 395 269
1874 Albany Baltimore Boston Chicago Cincinnati Cleveland Detroit Louisville Milwaukee New Orleans New York	Banks 7 14 58 19 5 6 3 9 4 7 48	Due from Redeeming Agents 4,415 2,166 9,770 4,907 1,539 903 743 407 463 209	Due from Other Banks 2,999 816 3,249 2,925 783 466 626 707 234 285 14,604	Due to Other National Banks 2,321 1,577 20,075 7,247 2,546 265 452 896 395 269 65,384
1874 Albany Baltimore Boston Chicago Cincinnati Cleveland Detroit Louisville Milwaukee New Orleans New York Philadelphia	Banks 7 14 58 19 5 6 3 9 4 7 48 29	Due from Redeeming Agents 4,415 2,166 9,770 4,907 1,539 903 743 407 463 209 - 5,534	Due from Other Banks 2,999 816 3,249 2,925 783 466 626 707 234 285 14,604 3,326	$\begin{array}{r} \mbox{Due to Other} \\ \mbox{National Banks} \\ 2,321 \\ 1,577 \\ 20,075 \\ 7,247 \\ 2,546 \\ 265 \\ 452 \\ 896 \\ 395 \\ 269 \\ 65,384 \\ 6,750 \end{array}$
1874 Albany Baltimore Boston Chicago Cincinnati Cleveland Detroit Louisville Milwaukee New Orleans New York Philadelphia Pittsburgh	Banks 7 14 58 19 5 6 3 9 4 7 48 29 17	Due from Redeeming Agents 4,415 2,166 9,770 4,907 1,539 903 743 407 463 209 - 5,534 1,579	Due from Other Banks 2,999 816 3,249 2,925 783 466 626 707 234 285 14,604 3,326 861	$\begin{array}{c} \mbox{Due to Other} \\ \mbox{National Banks} \\ 2,321 \\ 1,577 \\ 20,075 \\ 7,247 \\ 2,546 \\ 265 \\ 452 \\ 896 \\ 395 \\ 269 \\ 65,384 \\ 6,750 \\ 1,335 \end{array}$
1874 Albany Baltimore Boston Chicago Cincinnati Cleveland Detroit Louisville Milwaukee New Orleans New York Philadelphia Pittsburgh San Francisco	Banks 7 14 58 19 5 6 3 9 4 7 48 29 17 2	Due from Redeeming Agents 4,415 2,166 9,770 4,907 1,539 903 743 407 463 209 - 5,534 1,579 98	Due from Other Banks 2,999 816 3,249 2,925 783 466 626 707 234 285 14,604 3,326 861 404	$\begin{array}{c} \mbox{Due to Other} \\ \mbox{National Banks} \\ 2,321 \\ 1,577 \\ 20,075 \\ 7,247 \\ 2,546 \\ 265 \\ 452 \\ 896 \\ 395 \\ 269 \\ 65,384 \\ 6,750 \\ 1,335 \\ 157 \end{array}$
1874 Albany Baltimore Boston Chicago Cincinnati Cleveland Detroit Louisville Milwaukee New Orleans New York Philadelphia Pittsburgh San Francisco St. Louis	Banks 7 14 58 19 5 6 3 9 4 7 48 29 17 2 7	Due from Redeeming Agents 4,415 2,166 9,770 4,907 1,539 903 743 407 463 209 - 5,534 1,579 98 835	Due from Other Banks 2,999 816 3,249 2,925 783 466 626 707 234 285 14,604 3,326 861 404 559	$\begin{array}{c} \mbox{Due to Other} \\ \mbox{National Banks} \\ 2,321 \\ 1,577 \\ 20,075 \\ 7,247 \\ 2,546 \\ 265 \\ 452 \\ 896 \\ 395 \\ 269 \\ 65,384 \\ 6,750 \\ 1,335 \\ 157 \\ 1,718 \end{array}$
1874 Albany Baltimore Boston Chicago Cincinnati Cleveland Detroit Louisville Milwaukee New Orleans New York Philadelphia Pittsburgh San Francisco St. Louis Washington	Banks 7 14 58 19 5 6 3 9 4 7 48 29 17 2 7 4	Due from Redeeming Agents 4,415 2,166 9,770 4,907 1,539 903 743 407 463 209 - 5,534 1,579 98 835 214	$\begin{array}{c} \text{Due from} \\ \text{Other Banks} \\ 2,999 \\ 816 \\ 3,249 \\ 2,925 \\ 783 \\ 466 \\ 626 \\ 707 \\ 234 \\ 285 \\ 14,604 \\ 3,326 \\ 861 \\ 404 \\ 559 \\ 97 \end{array}$	$\begin{array}{c} \mbox{Due to Other} \\ \mbox{National Banks} \\ 2,321 \\ 1,577 \\ 20,075 \\ 7,247 \\ 2,546 \\ 265 \\ 452 \\ 896 \\ 395 \\ 269 \\ 65,384 \\ 6,750 \\ 1,335 \\ 157 \\ 1,718 \\ 103 \end{array}$

Table 1: Summary of the balance sheets of reserve city banks by city in 1873 and 1874

Note: The table reports the number of banks in each reserve city along with selected balance sheet quantities, aggregated at a city level, in 000's of US dollars from the *Annual Reports of the Comptroller of the Currency* in 1873 and 1874. New York, as the central reserve city and ultimate redeeming agent, does not have any redeeming agent balances.

the data do not allow us to measure the actual quantities of interbank balances transferred between institutions or cities. Rather, we observe only the aggregate holdings of each bank, so if a bank were both to gain interbank deposits from some counter-parties and lose them from others we would know only the net change. We can also identify a lower bound for actual interbank deposit transfers from the available data by looking at the sum of absolute changes in the "due to other national banks "balance sheet entry across all banks. This figure, divided by two to account for deposits going out of one bank and into another, puts the minimum level of change at over \$25m. It is important to emphasize that this is a minimum and in reality the change would have been much larger.

2.2 Network Construction

A bank that deposits funds in another creates a connection between the two institutions, and across the system these linkages form a network in which the nodes are banks and the edges are interbank deposits. The balance sheets from the Comptroller's reports include the total amount of interbank deposits held by and owed to each bank.⁹ Given that a comprehensive set of individual linkages in 1873 is not available, we build a representative network using the bank-level aggregates, regulatory structure, and a set of plausible bank

$$L^{j} = L^{j}_{*} \frac{\sum_{i=1}^{M} (A^{i}_{R} + A^{i}_{B})}{\sum_{i=1}^{M} L^{i}_{*}},$$
(1)

⁹For a network constructed from our data to be valid technically, the total interbank deposits held by a given bank must equal the "Due to national banks" entry on the liability side of the balance sheet. Similarly the amount a bank keeps in reserve city banks and other banks must equal the sum of the amounts "Due from redeeming agents" and "Due from other national banks" on the assets side. By definition the total amount "Due to" across the system should therefore equal the total of the "Due from" values as every deposit appears as an asset for one bank and a liability for another. The data, however, do not quite conform to this standard, with the total asset position larger than the liability position by about 2 percent. This likely reflects deposits in transit between banks. When funds are transported from a creditor to a debtor via non-instantaneous means they appear on the asset side of the creditor when they depart and on the liability side of the debtor only when they later arrive. If the money travels in the opposite direction it is removed from the debtors liabilities on departure and added to the creditors assets upon arrival. We resolve this by scaling each bank's interbank liabilities as follows:

where M is the number of banks, L_*^i is the recorded value of interbank liabilities of bank i, and A_R^i and A_B^i are the interbank assets due from redeeming agents and other banks respectively. The result is that total interbank liabilities are set equal to total interbank assets while preserving the relative sizes of these positions across banks. We then construct valid interbank networks with the scaled data.

utility functions for choosing correspondents and interbank deposit amounts.¹⁰

The Comptroller's reports include the gross interbank positions of each bank but do not include the individual banks in which the interbank deposits are placed. The network we construct can be viewed as estimating this missing information using a matrix where rows and columns are banks and entries correspond to specific interbank deposits. In this setting, the "Due to national banks," "Due from redeeming agents," and "Due from other national banks" entries provide three constraints on the matrix entries for each bank. With diagonal entries set to zero by definition, a system with M banks has 4M constraints to estimate M^2 variables, leaving the problem under-constrained for any network with M > 4 and therefore allowing an infinite number of valid networks. Not all of these networks, however, are realistic. For example, a network could be constructed where each bank places deposits in the most distant banks, yet the costs of transporting funds and gathering information would make such a network extremely unlikely in practice. To assess the likelihood of various networks, we therefore model banks' preferences for given deposit holding institutions through a common utility function, F(i, j). Indeed, the consideration of bank preferences is the key difference between our method of building the network and a maximum entropy approach. We then identify networks that maximize utility across the system:

¹⁰The practice of building networks from limited data has received much interest in the literature on systemic risk analysis. The most common technique is the maximum entropy approach, which makes a minimal set of assumptions about the unknown network. This approach, however, has its challenges (see, for example, Mistrulli 2011), and has led to attempts to find alternative approaches (see Basel Committee on Banking Supervision 2015 for a comparison of various methods). Given the structural information about the National Banking System that is available to us, we are not limited to a maximum entropy approach and can potentially achieve superior estimates of the underlying network. In this sense our approach has similarities with that of Elsinger et al. (2006) who use sectoral information among banks to inform their estimates of the Austrian interbank market.

$$max \sum_{i=1}^{M} \sum_{j=1}^{M} F(i,j)x_{ij}$$
(2)

subject to

$$\sum_{i=1}^{M} x_{ij} = L^j \qquad \qquad \forall j = 1, \dots, M \tag{3}$$

$$\sum_{j=1}^{M} x_{ij} R(i,j) = A_R^i \qquad \forall i = 1, ..., M$$
(4)

$$\sum_{j=1}^{M} x_{ij} (1 - R(i, j)) = A_B^i \qquad \forall i = 1, ..., M$$
(5)

$$x_{ii} = 0 \qquad \qquad \forall i = 1, \dots, M \tag{6}$$

where x_{ij} is the size of deposit placed in bank j by bank i and R(i, j) is an indicator function equal to unity if bank j could be a reserve agent of bank i and zero otherwise.¹¹ The optimal network clearly depends on F(i, j), i.e., the utility of bank i placing deposits in bank j. Below we describe a set of preferences over potential deposit locations that generate plausible networks.

The first factor governing preferences is distance. While wire transfers by telegraph began with Western Union's service in 1871, they were little used in 1873 and 1874, and net settlements typically required money to move physically by foot, horse, wagon, rail or canal, each of which entailed a cost. Distance also meant that banks had less timely and complete information about distant counter-parties than for those nearby. These factors led banks to favor more proximate interbank partners.¹²

We measure the distance between banks based on the available transport routes. Maps of canals, navigable rivers and rail networks present in 1870 are from Atack (2013). We also add sea routes between locations along coastal waters in a similar manner to Donaldson

¹¹For country banks the set of possible redeeming agents includes all banks in reserve cities and in the central reserve city of New York. For reserve city banks the set includes only New York banks, and for New York banks the value is always zero.

¹²Such proximity effects persist in today's banking systems, for example, Degryse and Ongena (2005) relate transportation costs to price discrimination in modern loan markets.

and Hornbeck (2016).¹³ We assume the existence of straight line roads traversable by horses, and while this is not completely realistic, the relative costs of land transport were so prohibitive that it was generally used for only short distances where the costs of a linear route would not differ much from the actual cost. Travel times for each transport type relative to horses are based on speeds estimated in (Boyd and Walton, 1972, pp. 246) and Kaukiainen (2001).¹⁴ We identify the shortest travel time between each pair of banks in the system using combinations of road, rail, canal, river and sea.¹⁵

The second component governing a bank's preferences is the soundness of the receiving bank. Since placing deposits in another bank presents a credit risk, banks may prefer to place deposits in counter-parties with strong balance sheets. There are numerous potential measures of financial soundness including capital, liquidity and various financial ratios. We analyze a wide set of these measures to identify the best representation of how banks behave in choosing correspondents and placing deposits with them.¹⁶

Let F(i, j) be the utility of bank i placing a single dollar of deposits in bank j:

$$F(i,j) = \frac{S(j)}{D(i,j)} \tag{7}$$

where S(j) is a function of the quality of bank j as measured by some balance sheet

¹³This is done by densely populating the coastal regions with possible start and finish points for sea routes and then connecting all pairs via the shortest possible sea route. Without detailed information for all actual sea routes in existence in 1873, this may overestimate the possible connections. Given, however, the generally convex area of the United States in which banks were located and the relative speed of sea versus other forms of transportation, the effects of measuring sea routes in this manner are minor.

¹⁴Boyd and Walton state the average speed of a train was 20 miles per hour in 1860 and 40 miles per hour in 1890, so we take 30 miles per hour as an average speed for 1873. Similarly they use an average water speed for canals and rivers of 4 miles per hour (although Kaukiainen suggests this could be as high as 10 miles per hour). Stage coach travel was faster than water travel but slower than rail and estimated at 6 miles per hour by Boyd and Walton.

¹⁵For numerical tractability we break rail, road and canal routes into 10 kilometer intervals at which routes may join, depart or continue along the same medium. We focus on speed and take the view that a single individual is traveling the route, therefore assuming changes in the mode of transportation to be costless. This decomposition generates more than 16,000 nodes in the transport network - approximately 2,000 correspond to banks and 14,000 to intermediate points. Using Dijkstra's algorithm we then efficiently calculate the approximately 250 million possible quickest routes in the transport network to identify the shortest distances between all pairs of banks.

¹⁶Section A in the appendix considers extensions to this function which may have affected banks' choices. This includes payments of interest by New York City banks on some of their correspondents' balances.

quantity or ratio, and D(i, j) is a function of the distance in kilometers between bank iand bank j. As such, bank i's utility from making an interbank deposit increases in the quality and proximity of the recipient. Both S(j) and D(i, j) are of the form:

$$S(j) = (\log(s_j + 2))^{\nu}$$
(8)

$$D(i,j) = (\log(d_{i,j}+2))^{\delta}$$
(9)

where s_j is the balance sheet quality of bank j and $d_{i,j}$ is the measured distance between banks i and j. The ν and δ parameters control the shape of the utility function. The +2 in the utility functions avoid instances where S(j) or D(j) would otherwise be undefined or less than or equal to zero.¹⁷

If the actual interbank network were known, the values of ν and δ could be estimated to fit the data most closely, but we do not observe the network directly and therefore use the available interbank aggregates to identify the most likely bank decision function and the optimal deposit network simultaneously. To estimate the network, we focus on those deposits mandated by regulation (i.e., the allocation of country banks' deposits to redeeming agents). Although country banks were required to hold deposits in reserve cities, there were no restrictions on which reserve city or reserve city banks (including those in New York) they could choose. We therefore evaluate the network based on the fit of interbank connections across redeeming agents based on their soundness and distance. In contrast, deposits among local country banks could be based more on idiosyncratic choices or short-term liquidity needs.

To assess a given utility function we exploit the freedom of banks to choose their reserve agent. We compare two allocations—with and without constraints. In the unconstrained allocation, each country bank places deposits in the reserve city bank that yields the highest utility, i.e., the reserve agent for which $\frac{S(j)}{D(i,j)}$ is maximized. When calculated for all banks, this delivers an optimal distribution of deposits across redeeming agents for the given utility function and set of parameter values. Importantly, this distribution does not

¹⁷More complex functional forms were considered, for example $\frac{\nu_1 S(j)^2 + \nu_2 S(j) + \nu_3}{\delta_1 D(i,j)^2 + \delta_2 D(i,j) + \delta_3}$, but had little effect on the results.

take into account the constraints imposed by the balance sheets themselves—in particular that the sum of deposits placed in a given bank must equal the value of its "Due to national banks" entry (Constraints 4 and 5 defined above).

In the constrained allocation, we include these "adding-up" restrictions when computing the network that maximizes total utility. If banks' optimal choices of redeeming agents and allocation amounts in the constrained network differ from those in the unconstrained, this indicates that the utility function is not fully capturing bank behavior, i.e., a given bank would prefer to place its deposits in a different redeeming agent but is unable to do so in the constrained case because that agent's "capacity" is absorbed by other banks. Since U.S. banks at the time were free to choose any redeeming agent, the two allocations would be identical only if the utility function perfectly captures bank behavior.

2.3 Implementation

We use a minimum distance approach in fitting the model to the observed data. The distance is defined as the sum of the squared differences between the amount each bank would place in its preferred reserve city when unconstrained and the amount placed when constrained. We use numerical optimization with a grid search of the feasible parameter space to find the optimal ν and δ that minimize this distance. A simple optimization reveals a highly uncorrelated parameter space (i.e., good solutions are not close together), which indicates sensitivity of the utility function to parameter choices. This is not due to the utility function itself, but rather to the highly non-linear nature of networks and their creation processes. Slightly different parameters can affect a bank's choice of where to place deposits, and as a consequence of the adding-up constraints, the optimal choices made by many others. In other words, even though the utility function retains its functional form, how precisely the various parameters lead it to rank individual pieces of information may differ slightly. To ameliorate this sensitivity, we take a Monte Carlo approach, repeating the optimization 100 times, adding each time a small amount of noise to the ν and δ parameters separately and independently for each bank, and then

average across the runs.¹⁸ This generates a smooth, correlated search space with a single minimum.

Given a particular utility function and the formulation defined above, the creation of the optimal network becomes an instance of the linear transportation problem, which has been well studied in operations research. This problem specifies a set of sources and sinks with known capacity. The sources and sinks are connected by links associated with a cost for each unit transported along them. The problem is normally specified as one of cost minimization, but we convert it to one of utility maximization. Sources are banks which act as interbank depositors, and their capacities are the amounts of interbank assets on their balance sheets. Similarly, sinks are receiving banks with capacities equal to their interbank liabilities. The cost of a link is the utility gained by the source bank for sending one unit of deposits to the connected sink. The solution to this problem is a network that maximizes total utility.¹⁹

In creating the network, we modify the utility function slightly to account for banks that are on average closer to other banks. If a network is created to maximize total system utility, banks situated closer to other banks, such as those in the northeast, would be favored, while little weight would be placed on the preferences of isolated banks, such as those in the northwest. This means that a northeastern bank could receive its first choice correspondent over a second choice with nearly equal utility, even if this results in a large relative reduction of utility for a northwest bank, so long as total system utility increases.²⁰ To resolve this, we normalize each bank's utility for placing deposits in a

²⁰This can occur because distance enters into the denominator of the utility function, i.e., all correspondent choices have relatively low utility for distant banks whereas differences of only a few kilometers can

¹⁸We search the grid in increments of 0.1 for both δ and ν . For each bank, we perturb each parameter by adding a value drawn independently and at random from $N(0, 0.02^2)$. We obtain similar results when all banks are given the same perturbation in each run.

¹⁹A bank may be both a depositor and receiver of interbank funds, and to prevent the algorithm from having a bank place deposits in itself (i.e., the shortest distance), the utility of these self links is set to zero. Similarly, a bank may have deposits in both reserve city and non-reserve city banks, as shown by separate listings on the balance sheet. To address this we represent each bank as two sources—one with capacity equal to the 'Due from redeeming agents' entry, which has zero utility for connecting to nonreserve city banks, and a second with the 'Due from other national banks' entry, which has zero utility for deposits in reserve city banks. For reserve city banks their 'Due from redeeming agent' deposits must be placed in New York banks, therefore the utility of placing these in any other bank is zero. After identifying the optimal network these sources are then recombined into the individual bank.

given bank by the sum of the possible utilities of depositing in all other M banks, noting once again that F(i, i) = 0:

$$\bar{F}(i,j) = \frac{F(i,j)}{\sum_{k=1}^{M} F(i,k)}.$$
(10)

Normalizing utility in this manner treats each bank's preferences as equally important in the allocation, and the \overline{F} values now correspond to the banks' relative preferences. The maximization is therefore maximizing the total relative preferences of banks across the system.

3 The Pre-Crisis Network

3.1 Specifying the Utility Function

To identify any optimal pre-crisis network it is first necessary to specify the utility function. In this section we test ten different measures of bank quality (S) to determine the specification that minimizes the error in forming a plausible network, with each specification implicitly testing a different utility model of how banks make decisions. The first measure we consider is the amount of specie on hand. Specie was the ultimate form of liquidity at the time and high levels would indicate an ability of the holding bank to repay deposits. The second and third utility functions explore a preference for large banks as measured by their total assets or total loans, reflecting the view that larger banks may be more established and therefore also have stronger reputations. The capital stock (and capital stock plus surplus fund) capture a similar view but now measure size and reputation by a bank's ability to absorb losses. The inverse of a bank's interbank exposures raises the possibility that banks avoid choosing correspondents that are vulnerable to potential instabilities among their correspondents. The final four measures are ratios: the ratio of specie to assets reflects the extent of available liquidity relative to balance sheet size; the ratio of specie to deposits reflects vulnerability to a run on deposits; and the

have large effects on utilities in areas where banks concentrate, such as the northeast. The shorter distances also mean these banks weigh more heavily in the system utility being maximized, thereby making the preferences of distant banks less important.

	Optimal	Optimal	Squ. Err.	Std. Err.
Bank quality (S)	ν	δ	$ imes 10^{12}$	$\times 10^9$
Specie	0.9	0.5	0.83	2.94
Loans	0.4	0.1	1.05	8.14
Assets	1.4	0.4	1.06	4.00
Capital stock $+$ surplus fund	1.8	0.5	1.21	3.29
Capital stock	0.6	0.6	1.28	2.72
Specie/assets	2.0	0.2	1.31	6.10
Specie/deposits	1.1	0.1	1.38	10.2
(Capital stock + surplus fund)/loans	1.8	0.1	1.60	3.42
(Capital stock)/loans	2.0	0.1	1.63	3.46
1/(interbank exposure)	0.0	0.5	1.82	4.44

Table 2: Closeness of fit between the model and observed interbank balances for 10 measures of bank quality (S).

Note: The table reports the average squared errors for optimized utility functions using ten different measures of bank quality. The squared error is defined as $(\sum_{i=1}^{N} (D^* - D_{Model})^2)$ where N is the index over banks, D_* and D_{Model} are the values of interbank deposits placed in bank *i*'s preferred reserve agent in the unconstrained (D^*) and constrained (D_{Model}) cases, respectively. Standard errors are computed across the squared errors from the 100 Monte Carlo runs. The bank quality (S) measures include specie, loans, assets, capital stock plus surplus, and capital stock as defined directly in the balance sheets. Interbank exposures are the sum of the "Due from redeeming agents" and "Due from other banks" entries. The ratio of specie to assets is defined as $\frac{\log(2+Specie)}{\log(2+Assets)}$, with the remaining functions defined analogously. Deposits are the sum of individual deposits, U.S. government deposits, interbank deposits and deposits of distributing officers.

ratios of capital (and capital stock plus surplus) to loans measure the ability of a bank to absorb adverse shocks requiring loan liquidation.

Using balance sheet data for all national banks on 12 September 1873, we compute the optimal parameter values and squared error for each of these objective functions and report the results in Table 2. Measuring bank quality as the amount of specie on hand generates the smallest error and the optimal parameter values for ν and δ are 0.9 and 0.5 respectively. Figure 1 shows the errors for all combinations of parameters using this function, and clearly indicates a region in the parameter space where the model consistently does well (errors are small) and where the optimal parameters fall. The squared dollar error of the average network for this parametrization across 100 Monte Carlo runs is 8.3×10^{11} . Given that the sum of the squared balance sheet entries for the "Due from redeeming agents" item is 1.54×10^{13} , the error is approximately 5% of the size of interbank positions placed in reserve cities (excluding banks in New York City).



Figure 1: Squared errors for different combinations of ν and δ .

Note: Each point on the surface depicts the average error across 100 Monte Carlo runs, with the lowest point corresponding to the minimum. Utility is calculated using specie as the measure of bank quality.

In all cases the errors associated with alternative objective functions are larger than those achieved with specie. This suggests that banks placed considerable weight on a counter-party's holdings of liquid assets and therefore on its ability to repay deposits when choosing correspondents. The next best performing functions are those related to measures of bank size, but even though these measures may produce reasonable networks the associated squared errors are more than 25 percent larger. While the idea that banks avoid other banks with large interbank positions also seems plausible, the results, with the optimal parameter of zero for ν , indicate that banks did not use this information in forming correspondent relationships. The weak explanatory power of this quality measure is likely because many of the largest banks, and especially those in New York, remain attractive as receivers of interbank deposits due to other factors even though they maintain large deposits in other banks.²¹ Based on these results, we use the amount of specie as our

²¹As robustness checks, we also estimate the network using the amounts "Due to national banks" and liquid assets (the sum of "checks and cash", "bills of other banks", "fractional currency", "specie", "legal tender" and "certificates of deposit") as measures of bank quality. Interbank deposit liabilities

preferred measure of balance sheet quality in subsequent calculations.

3.2 Choice of Reserve Agents

Using the optimal utility function with specie and the parameter values shown in the first row of Table 2, Figure 2 shows the primary reserve agent for each bank as computed by the network algorithm and rules for a single Monte Carlo simulation. The primary reserve agent is the agent in which a given bank places the most deposits. We present a single simulation rather than an average over multiple simulations to illustrate the geographic mix of choices that the network can spawn rather than modal choices, which tend to overstate the number of banks placing their highest shares of deposits in New York. To promote readability the figure focuses on three of the reserve cities only, New York, Chicago and Boston, with banks placing their reserves in other cities falling under the 'Other' grouping. Figure 3 offers a closer view for the mid-Atlantic area, where banks depositing in Baltimore, Philadelphia, and Washington, DC can be distinguished more easily. Although Figure 2 and Figure 3 cannot distinguish all banks in the various clusters due to their density and overlap, the key finding is that banks near a reserve city tend to use banks in that city as the primary reserve agent, but that the geographic influence of each reserve city is based on a combination of specie reserves, distance from other reserve cities, information costs, and the density of banks in the region. The exception is New York City, which was a popular reserve agent for country banks from all regions, including banks in locations far from any reserve city.²²

Figure 2 also shows that the various reserve cities primarily place deposits in New York. The clearest cases for observing this are Chicago, New Orleans and San Francisco.

generate inflows of cash which must then, at least in part, be maintained as liquid assets. The model could, in principle, therefore be identifying those banks which have large interbank deposit positions on their balance sheets as the most attractive destinations for interbank deposits. These measures, however, produced significantly higher errors indicating that this is not the case, but rather that the model specification based on specie is identifying a relationship between the behavior of banks and their attractiveness as counter-parties.

²²This occurs because distance is less important to the optimal choice of reserve agent for an isolated bank with all possible reserve agents relatively distant. This makes specie on hand, for which the New York banks generally held the most, the primary determinant of correspondent choice.



Figure 2: Reserve cities where individual banks in the model place a majority of interbank deposits.

The shade of each circle denotes the reserve city receiving the largest share of a given bank's deposits in the model. Results are from a single Note: Reserve city categories in the figure are limited to New York, Boston, Chicago, and "other" reserve cities for expositional clarity. simulation of the model and are representative of other simulations.



Figure 3: Detailed view of Figure 2 showing the Northeastern United States.

Note: The shade of each marker denotes the reserve city receiving the largest share of a given bank's interbank deposits in the model. Results are from a single simulation and are representative of other simulations.

4 Fit of Model Predictions to 1874 Data

In this section we impose a set of structured withdrawals on the optimal network formed with the data from 12 September 1873, and compare the results with the observed distribution of interbank deposits on 2 October 1874, a year after the crisis. Each scenario tests a different model for how the crisis may have propagated.

4.1 Withdrawal Patterns and Processing Rules

We consider the following nine cases:

 No withdrawal by any bank, leaving the 1873 distribution of interbank deposits unchanged. Since this allows for a direct comparison of observed interbank deposits in 1874 and 1873, it serves as a baseline for quantifying the effects of the various structured shocks we consider.

- 2. All banks reduce their 1873 interbank deposit assets by the same percentage α_2 .
- 3. All banks reduce their 1873 interbank deposit assets, but the extent depends on the distance of their counter-parties from New York City. In other words, a given bank's correspondents further from New York see their interbank liabilities reduced less than those nearer to New York. This could reflect concerns among depositing banks about the vulnerability of linkages between their own counter-parties and the New York banks that were the source of the crisis. Interbank positions are reduced by f(i) where $f(i) = \alpha_3 + \beta_3(1.0 - \frac{d_i^{NY}}{max_j d_j^{NY}}))$, and d_i^{NY} is the economic distance between a given bank's receiving correspondent and New York.
- 4. All banks with deposits in New York City, as indicated by the network, withdraw a fraction γ_4 of their interbank deposit assets. This simulates a more concentrated shock and withdrawal from the New York banks.
- 5. The same withdrawal as 4 but with an additional reduction in the interbank deposits of all non-New York City banks by α_5 analogous to 2.
- 6. The same withdrawal as 4 but with an additional reduction in interbank deposits by all banks dependent on the receiving bank's location in a manner analogous to 3 and scaled by variables α₆ and β₆.
- 7. All banks with interbank deposits in New York withdraw a fraction $\gamma_7(=\gamma_4)$ of their funds as described by shock 4 and place them in other reserve city banks with the reallocation maximizing their respective utilities. This once again simulates a more concentrated panic.
- 8. The same withdrawal and reassignment as 7 but with an additional reduction in interbank deposits by all non-New York City and non-reserve city banks of α_8 in a manner analogous to 2.²³

 $^{^{23}}$ We restrict this withdrawal to country banks as they were not the primary focus of the withdrawal from New York or the reassignment to reserve cities.

9. The same withdrawal and reassignment as 7 but with an additional reduction in the loans of all banks dependent on the receiving banks' proximities to New York in a manner analogous to 3 and scaled by variables α_9 and β_9 .

For each of these cases, we numerically find the applicable α and β parameters which minimize the mean of the squared errors at the county level between the observed balances "Due to national banks" in the October 1874 data and the interbank liabilities generated by the shocked model for 1873. Of the nine cases, the first three use observed balance sheet values from 1873 to make the comparisons²⁴; the remainder draw the comparisons by propagating the various shock through the 100 Monte Carlo networks we constructed for 1873. Since many banks under the algorithm make small changes to minimize the countywide error and which likely differ from changes that would be observed in practice, the results are best considered as a probability distribution, i.e., the probability that a bank will respond in the manner the model predicts.

4.2 Observed Changes in Interbank Deposits, 1873 to 1874

Figure 4, corresponding to Case 1, shows observed changes in interbank deposits by county between the Comptroller's reports of 1873 and 1874. Darker shades correspond to counties with larger increases in interbank liabilities, lighter shades to those losing balances, and unshaded areas to those without a national bank. The map shows the majority of reductions occurring in the northeast, and particularly in New York City and nearby counties. The national banks in New York alone see a reduction of \$6 million, which is 8.6 percent of their interbank holdings in 1873. Reductions are generally smaller and more isolated in other regions, but some counties see significant reductions, and in particular those including the reserve cities of Baltimore (\$1.41 mil. or 47.3 percent), Cincinnati (\$632,000 or 19.9 percent), Philadelphia (\$275,000 or 3.9 percent), and Pittsburgh (\$211,000 or 13.6 percent). Many counties also see increases, including the reserve cities of Chicago (\$1.13 mil. or 18.5 percent), Detroit (\$130,000 or 40.4 percent), Louisville (\$485,000 or

²⁴The total deposits in each counterparty are given by the balance sheet whilst the locations are fixed, the individual bank deposit relationships are therefore not needed to calculate the reductions in these cases.



Figure 4: Changes in interbank deposit liabilities by county, 1873 to 1874.

Note: Dollar amounts are aggregated from the 'Due to other national banks' entries for 12 September 1873 and 2 October 1874 in the Annual Report of the Comptroller of the Currency.



Figure 5: Magnified view of Figure 4 for the Northeastern United States.

117 percent), and San Francisco (\$135,000 or 600 percent). Figure 5 provides a closer view of the northeast and underscores the large reductions experienced by most counties in the region, particularly those close to New York along with the reserve cities of Albany (\$227,000 or 8.9 percent) and Baltimore (\$1.41 mil. or 47.3 percent). Boston is an important exception, however, with interbank deposits rising by more than \$4 million (25 percent), which is the largest gain of any county in the United States over the one-year period.

In the next section we compare the observed changes in interbank deposits by county to those generated by our network model of the crisis to gain insights about possible paths through which these changes occurred.

4.3 Comparing Simulated to Observed Interbank Balances in 1874

The mean squared errors, presented in Table 3, provide similarity measures between the 1874 data and the distributions of interbank deposits we obtain by simulating the nine

Scenario	Dollar Squared Error (10^{13})	Standard Error	γ_i	α_i	β_i
1	8.57	5.77×10^{6}	-	-	-
2	3.38	1.41×10^6	-	0.09	-
3	3.26	$3.86 imes 10^6$	-	0.00	0.10
4	2.24	1.45×10^7	0.08	-	-
5	2.24	1.45×10^7	0.08	0.00	-
6	2.10	1.46×10^7	0.08	0.00	0.45
7	1.68	2.08×10^9	0.08	-	-
8	1.53	2.08×10^9	0.08	0.44	-
9	1.53	2.08×10^9	0.08	0.00	0.45

Table 3: Closeness of fit between model predictions and observed interbank deposit liabilities in 1874 by county.

Note: Results are shown for each of the nine cases described in Section 4.1. Errors are based on differences between predicted values for interbank deposit liabilities in 1874 and the observed 'Due to national banks' entries in the 1874 Annual Report of the Comptroller of the Currency for all counties with one or more national banks. The squared error is defined as $(\sum_{i=1}^{N} (D_{Model} - D_{1874})^2)$ where *i* is the index over counties, and D_{Model} and D_{1874} are the model predictions and observed data summed at the county level, respectively. Standard errors for each case are calculated from the squared errors across 100 Monte Carlo runs.

cases described above using the data and derived network for 1873. Case 2 represents the simplest withdrawal pattern: a flat reduction of interbank deposits across all banks, ignoring the identify of the depositing bank, the receiving bank, and their locations. The optimal value of α_2 is 0.09. Simulating a withdrawal of this magnitude reduces the error by approximately 61% relative to the baseline case (1) of "no withdrawals," and thus provides a more accurate matching between balances generated by the model and the observed interbank positions. While it significantly reduces the error, a flat reduction of interbank deposits of 9% across the system clearly cannot account for the increases in interbank deposits observed for some reserve cities.

Case 3 models a more sophisticated withdrawal. In this setting, banks closer to New York City are presumed to be at greater risk of failure because of greater exposure to the potentially risky investment practices of the New York banks. The exposure may be either direct through actual deposits in New York banks, or indirect, through connections to exposed banks. This gradient approach, with parameters $\alpha_3 = 0.00$ and $\beta_3 = 0.10$, reduces the squared error another 1.4 percent from the flat withdrawal pattern. As such, whilst the concentration of deposits in the New York area mean that there is relatively little difference in terms of fit between this and the flat withdrawal the parameters indicate potential depositing banks viewed correspondents close to New York as having greater exposure to crisis conditions.

Case 4 models withdrawals from New York by all banks using it as a reserve city. The scale of the withdrawals is not set to minimize the squared error, but rather to match the total withdrawals of 8% from New York observed in the data. Accounting for these targeted withdrawals generates a 74% reduction in the error from the baseline. Unlike Case 3 above, however, a further flat withdrawal by all banks from all non New York City banks does not reduce the error further - the optimal flat withdrawal size is 0 (Case 5). This is because the model leaves several of the reserve cities with too few deposits after the New York withdrawal so that further reducing the overall level of deposits in the system actually worsens the fit. An additional distance based withdrawal, as in Case 6, does reduce the error by another 2%. This is because this withdrawal predominantly targets banks in counties close to New York that also saw a reduction in deposit positions during the crisis but were not included in the initial New York City-only withdrawal.

Case 7 permits non-reserve city banks to reallocate reserves withdrawn from New York to other reserve cities, but once reallocated they remain in the new reserve cities. These withdrawal rules reduce the squared error by 81% relative to the baseline, an improvement of 7 percentage points over Case 4. Even though no reserves are removed from the system, the reallocation from New York to other reserve cities through the utility function matches the empirically observed distribution closely.

Figure 6 shows the differences in interbank reserve balances between the model outcome for Case 7 and the observed county-level totals in 1874. The results closely match the actual increases observed for Chicago, Boston and Detroit. This, together with the improvement in fit, indicates that reserves were withdrawn from New York City banks by their interbank correspondents and placed in other reserve cities with large specie holdings. Importantly, it was not the behavior of banks in reserve cities that drove this dynamic as they did not reallocate their funds, but rather the country banks that moved funds out of New York and placed them in Chicago, Boston and Detroit. Without the network, we would be unable to theorize about where banks depositing in New York may have reallocated their funds. The only region where the model predictions for Case 7 significantly differ from the observed October 1874 balances are for banks in the counties immediately surrounding New York City, where there were clearly substantial withdrawals that the model does not pick up. The following two shocks, however, address this issue.



Figure 6: County-level differences in interbank deposit liabilities between the model predictions under Case 7 and the 1874 data for the Northeastern United States.

Note: Dollar amounts are aggregated from the 'Due to other national banks' entries in the 1874 *Annual Report of the Comptroller of the Currency*. Positive values indicate counties where the model overestimates interbank deposit liabilities.

Case 8 models an additional uniform reduction in interbank deposits after Case 7 for all country (i.e., non reserve city) banks. The utility maximizing additional withdrawal of 45% from country banks reduces the error by another 2 percentage points from the baseline. This configuration indicates that while reserves were being moved out of New York to other reserve cities there were also significant general reductions in interbank positions among country banks. The mechanism captured by case 7, whereby banks



Figure 7: County-level differences in interbank deposit liabilities between the model predictions under Case 9 and the 1874 data for the Northeastern United States.

Note: Dollar amounts are aggregated from the 'Due to other national banks' entries in the 1874 *Annual Report of the Comptroller of the Currency*. Positive values indicate counties where the model overestimates interbank deposit liabilities.

move deposits from New York to other reserve cities, leads to deposits in the reserve cities coming very close to those observed in the actual data. The general withdrawal modeled by Case 8 additionally captures this effect for other counties. The use of a more complex gradient-based model of withdrawal, as considered in Case 9 has a negligible effect on the error. This may be contrasted with Case 3 where the gradient function led to a significant further reduction in the error. In Case 9 the majority of this effect has already been included as a result of the network withdrawal and so has a minimal additional impact. This final aspect of the model, however, significantly improves the fit as shown in Figure 7, and it can be seen that the large observed reductions from banks in areas around New York city, including banks along the Connecticut coast, are now more closely captured.

5 Predicting Banks that Panic

In this section we identify the set of changes in the reserve agents of individual banks that best explain the transition from the county-level distribution of deposits in 1873 to those observed in 1874. We consider a bank that changes reserve agent as one that "panics" in response to the crisis and reallocates deposits to more desirable counter-parties. Our 1873 network predicts specific banks that would have switched redeeming agent, and a comparison of how well these line up with actual changes in geographic balance sheet quantities provides a test of the model and the validity of the network.

5.1 Algorithm

We use the following algorithm to identify those banks most dissatisfied with their existing reserve partners in 1874 and therefore most likely to make changes.

- 1. Commence with the pre-crisis 1873 network.
- 2. Re-scale each bank's interbank deposits by $\frac{1874PositionSize}{1873PositionSize}$.
- 3. Calculate for each bank the relative change in utility for transferring its deposits from its current reserve city to every other reserve city.
- 4. Sort the potential transfers in decreasing order of utility improvement.
- 5. Make transfers by moving an amount of deposits K from bank i in reserve city m_s to a bank in reserve city m_d , where $K = max(0, min(D_{m_s} - K \ge C_{m_s}^{1874}, D_{m_d} + K \le C_{m_d}^{1874}, d_{m_s}^i)).$
- 6. Stop when for all reserve cities $m C_m^{1874} = D_m$

The re-scaling in step 2 is done on a deposit basis. Each position that a given bank has in another bank is multiplied by the factor above determined by the balance sheet data. This results in the total interbank assets of each bank being equal to the amount specified on the 1874 balance sheet. This allows us to control for banks' decisions of whether to increase or decrease their interbank deposit positions more generally and thereby focus on the re-allocation question. This re-scaling also ensures that total interbank deposit liabilities in the system are approximately equal to those recorded in 1874.²⁵ The next steps re-assign deposits away from reserve cities with too many interbank deposits (i.e., where the total deposits predicted by the network exceed the sum from the 1874 balance sheets) and to reserve cities with too few so that the model's city level balances line up with the 1874 data.

In the above d_m^i denotes the deposits placed in reserve city m by bank j according to the network after scaling. D_m denotes total deposits placed in reserve city m by banks according to the network, i.e., $D_m = \sum_{i=1}^n d_m^i$. C_m^{1874} is the total interbank deposits in reserve city m according to the 1874 balance sheets. The funds moved in step 5 are therefore the maximum amounts such that their movement does not lower total deposits in the reserve city from which they are moved below the amounts stated on the 1874 balance sheets, exceed the amount of funds on the 1874 balance sheets in the destination city, or exceed the amount of deposits the individual bank has available to move.

We apply the algorithm to each of the 100 networks generated initially by the Monte Carlo runs described in Section 3, and then identify those banks most likely to change reserve agent and the reserve agents to which they move.

5.2 Changes in Reserve Agents

Table 4 indicates how banks changed their primary reserve cities between 1873 and 1874. It does not include new banks that entered after the 1873 report. The results are broadly intuitive. We see that reserve cities which both gained and lost interbank deposits tended to exchange with other nearby reserve cities. Boston is interesting in that it capitalizes on New York's losses, but also attracts new interbank depositors from reserve cities further away than the others.

We can use the set of banks changing primary reserve city to characterize banks that "panic." Given that distances remain fixed, changing one's counter-party indicates a reduction in utility from remaining with the previous reserve agents, which in the model

 $^{^{25}\}mathrm{Note}$ as before there is a very small error here due to money in transit.

								Ne	w Rese	rve Ci	ty						
	Reserve City	Alb.	Bal.	Bos.	Chi.	Cle.	Cin.	Det.	Lou.	Mil.	N.O.	N.Y.	Phi.	Pit.	S.F.	St.L.	W_{as} .
	Albany	10.75	0	7.77	0	0	0	0	0.2	0	0	0	0	0	0	0	0
	Baltimore	0	32.91	23.48	0	0	0	0	2.64	0	0	0	0.26	0	0	0	0
	Boston	0	0	343.16	0	0	0	0	0	0	0	0	0	0	0	0	0
	Chicago	0	0	0	93.08	0	0	0	0	0	0	0	0	0	0	0	0
λη	Cleveland	0	0	2.36	0	1.09	0	0	4.97	0	0	0	0	0	0	0	0
\mathbf{C}	Cincinnati	0	0	0	0	0	64.61	0	6.12	0	0	0	0	0	0	0	0
элл	Detroit	0	0	0	0	0	0	6.65	0	0	0	0	0	0	0	0	0
əsə	Louisville	0	0	0	0	0	0	0	9.4	0	0	0	0	0	0	0	0
Я	Milwaukee	0	0	2.45	7.86	0	0	0	0.14	8.26	0	0	0	0	Ļ	0	0
en	New Orleans	0	0	8.91	0	0	0	0	0	0	5.27	0	0	0	1.41	0	0
ıSı.	New York	0	0	47.6	0.01	0	0	7.09	2.45	0	0	1000.68	0	0	1.6	0	0
IO	Philadelphia	0	0	0.04	0	0	0	0	0	0	0	0	66.5	0	0	0	0
	Pittsburgh	0	0	0.28	0	0	0	0	7.88	0	0	0	0	3.84	0	0	0
	San Francisco	0	0	0	0	0	0	0	0	0	0	0	0	0	2.28	0	0
	St. Louis	0	0	8.2	2.87	0	0	0	0	0	0	0	0	0	0.11	13.45	0
	Washington	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	1.37

Table 4: The average number of banks changing reserve city in which the majority of their interbank deposits are placed in the model and under the reallocation algorithm. Note: Diagonal values reflect the number of banks that do not change their primary reserve city. All values reflect averages across Monte Carlo runs of the 100 networks generated in Section 3. As a result of variation in the number of banks using each reserve city in the generated networks, the numbers of banks moving (or not moving) may not be whole numbers. can be driven by a weakening of the changing banks' balance sheets compared to others. A check of the model can therefore be obtained by comparing changes in actual balance sheet quantities, including those not used in the model, between 1873 and 1874 for banks where the model predicts a switch in primary reserve city against those for which the model does not. For this part of the analysis we exclude all reserve and central reserve city banks and focus only on country banks since reserve city banks are by definition unable to change their reserve city away from New York and New York banks have no reserve agent. Similarly we exclude banks without a reserve agent in 1873 or 1874, either because they were new, went bankrupt or did not place interbank deposits. A bank "panics" when its modal choice of reserve city across the 100 Monte Carlo networks changes. As discussed earlier, the model is best viewed as a probabilistic mapping, and by focusing on those banks changing their modal choice of reserve agent rather than any banks that change a reserve agent under any circumstances, we identify the set most likely to have panicked. We report the findings in Table 4.

The results provide a clear picture - those banks that the model predicts would panic show signs of panic in their balance sheets. Most significantly, banks that panic dramatically increase their specie holdings compared to those that do not (73% vs. 6%). This is consistent with our hypothesis that banks use specie holdings as a key determinant of financial quality. It should be highlighted that this is the specie present in the country bank – not the reserve agent – and therefore is not used in the switching algorithm described above. In other words, banks that our model predicts would change reserve agent do so because they are dissatisfied with their counter-party's specie holdings and, in addition to changing agents, choose to fortify their own specie positions in response. This increase in safe assets is accompanied by a simultaneous decrease in risky ones. Panicking banks reduce their interbank positions in reserve agents more than those that do not. Their increased perception of counter-party risk leads them to move cash from reserve agents to specie in their own vaults. At the same time panicking banks also show a marked increase in general aversion to risk, and decrease their lending to non-banks by a greater

Due to	Other Banks	-35.82%	-40.69%
Undividended	$\operatorname{Profits}$	-1.05%	-0.78%
	Assets	-0.69%	-0.99%
	Specie	72.72%	6.01%
Due from	Reserve Banks	-19.17%	-14.27%
	Loans	-2.45%	-1.38%
	Group	Changed	Not Changed

Table 5: Percentage changes in the observed mean balance sheet values of banks changing modal choice of reserve city in the model and those that do not between 1873 and 1874 Note: There are 159 and 1,475 banks in the 'Changed' and 'Not Changed' categories, respectively. The remaining banks in the system are excluded as either reserve or central reserve city banks, or due to having no reserve agents in 1873 and/or 1874. $\mathrm{amount.}^{26}$

Adjusting balance sheets to compensate for changing risk levels has consequences for the panicking banks. By reducing their risky asset positions and investing more in safe assets such as specie, panicking banks sacrifice profits to achieve greater resilience. Further, while both types of banks see a decline in interbank liabilities, the improved financial soundness of non-panicking banks results in smaller decreases than those observed for banks we predict to have panicked.

The strength of these results in supporting the model and our estimation approach should be emphasized. We have used interbank balances plus our theory of how interbank deposit decisions are made to generate a set of banks that the model predicts as prime targets for changing primary reserve city during the crisis. Examination of the balance sheets of these banks, and in particular data items not used by the model (e.g. loans, assets, and specie of the country bank), matches in nearly all cases what we would expect of banks that panic. In other words, we see a net change in reserve city deposits, use the model to predict which correspondent banks made these changes, and find that their actual balance sheets are consistent with these predictions. The predictions could not be made without the interbank networks we create, and these networks are what allow us to generate testable predictions that could not have been made or evaluated previously.

6 Stability

We now consider the propensity for bank failures to spread through the interbank deposit network, focusing on direct losses of banks due to the crisis rather than their actions in response to it. Our network provides a mechanism through which losses at a bank may leave it with insufficient funds to redeem interbank liabilities, causing its counter-parties to incur losses also. We separately consider two shocks - a liquidity shock and an equity shock. In both cases, since the 1873 crisis originated in New York, we simulate the losses that occur when the balance sheets of New York City banks are damaged. In the case

²⁶Overall balance sheet sizes change relatively little. Both sets of banks reduce their overall sizes, but this is less pronounced for the panicking banks, likely reflecting their increases in safe assets.

of the equity shock, loans made by New York banks are reduced by some percentage R, reducing the equity of a given bank and potentially forcing it into bankruptcy. For the liquidity shock, the total liquidity (i.e., specie and cash) available to each New York bank is reduced by R, potentially rendering it unable to make required payments to other banks. In both cases we vary R to compare the magnitude of the losses and the incidence of failure.

For the equity shock we assume that banks are initially solvent, i.e., they would be able to repay the deposits of creditor banks if their own interbank assets are repaid. In addition, each bank has capital and possibly undivided profits and a surplus fund to absorb losses before they fall upon its creditor banks. If these funds are insufficient, any losses are divided among the interbank creditors in proportion to their deposits with the troubled bank. Note in this case we deliberately make strong assumptions about the propagation of crisis: Banks must repay deposits immediately and all losses beyond equity are constrained to interbank depositors. Similarly we do not model the effects of clearinghouses within the system, which would also act to reduce losses. Despite these assumptions, as we will show below, the system is very stable.

The liquidity shock is similar, but rather than tracking bank equity the model measures a bank's ability to make payments from liquid assets. We assume that banks must repay their interbank creditors, and to do this they may withdraw their deposits from other banks to meet their obligations. If they are unable to make the payments, losses are divided between creditors in proportion to the size of position. Again our assumptions are relatively strong in that we assume all banks can potentially withdraw all of their cash at the same time, but again we will show that relatively few banks fail as a result.

To compute the effects of losses in New York, we employ the default algorithm of Eisenberg and Noe (2001). This approach identifies a unique and feasible set of interbank payment flows by considering each bank's payments due to and from other banks and their access to external resources. The external resources may be thought of as the bank's store of equity (liquidity) in the case of the equity (liquidity) shock from which it may draw to meet shortfalls in its required payments. The Eisenberg and Noe algorithm iteratively calculates a clearing vector from the set of payments made between banks until a fixed point is reached. The problem itself is not trivial due to the potential for cycles within the interbank market and the requirements that losses are divided proportionately among creditors and no bank makes payments greater than its available resources. Eisenberg and Noe show that, under fairly general assumptions, the algorithm finds a unique clearing vector. Banks unable to make their required payments from a combination of the payments they receive and their external resources are classed as insolvent and distribute their available funds to their creditors in proportion to the size of their debts.²⁷

We set the payments 'due from' and 'due to' other banks equal to those defined by the network. Each bank has external resources

$$e_{i}^{l} = ChecksAndOtherCashItems + BillsOfOtherNationalBanks + FractionalCurrency + Specie + LegalTenderNotes + USCertificatesOfDeposit$$
(11)

in the case of the liquidity shock and

$$e_i^e = Capital + SurplusFund + Profits + LoanLossReserve + DepositsDueToBanks - DepositsDueFromBanks$$
(12)

in the case of the equity shock.

In this latter case the final two terms, as previously noted, ensure that the bank has sufficient equity to make its payments if the banks own deposits are repaid, and correspond to the payments the banks will make to and receive from other banks - both redeeming and non-redeeming. Taken together with the positions directly due from and to other banks, these values ensure that a given bank's net position is equal to its equity. Without this, a bank with a net position due to other banks exceeding its own equity would be considered bankrupt even though in practice this could be offset by other positions on the asset side of its balance sheet (e.g. loans).²⁸ The shock to bank equity is equal to Rl_i for any New York bank *i*, where l_i is the amount of loans on bank *i*'s balance sheet, and is

²⁷See Appendix B for a full description of the algorithm.

 $^{^{28}}$ A similar adjustment is not required for liquidity since the algorithm tracks the movement of physical currency rather than a bank's net assets and liabilities and, dependent on the bank's investment decisions, there is no requirement for the two to balance.

zero for any non-New York City bank. In the case of the liquidity shock l_i is replaced by e_i^l , the bank's available liquidity in the above formula, and the reduction is carried out in the same manner. In both cases the shock may be thought of as reducing (or even making negative) the banks capacity to make payments and therefore increasing the likelihood of its inability to do so. For banks with a negative balance of payments to other banks the reduction may lead them to fail.

Figure 8 shows the number of banks that fail across the financial system for different magnitudes R of the separate equity and liquidity shocks. For either shock, even when large, it can be seen that very few banks fail. Liquidity shocks have larger effects for intermediate shock sizes, indicating a relatively greater sensitivity to a loss of deposits than of equity. The scope of the failures, however, is restricted in that few banks outside of New York City fail as a result of the liquidity shock. In other words, most banks outside of New York are able to recover enough of their deposits to prevent them from becoming illiquid. A similar effect is observed for equity shocks, which begin to generate significant failures only for shocks that are much larger than could have been reasonably expected during the crisis.

Taken together these results show that it is unrealistic to believe that direct losses from either liquidity or equity shocks were the main force behind the panic of 1873. Even under the largest shock and with assumptions likely to exacerbate the scale, the maximum number of failures from either the liquidity or equity shocks is equivalent to approximately 2% of the banks in the system. This contrasts with the results of previous sections which show that over 10% of eligible banks panicked during the crisis (Section 5) and 9% of interbank deposits were on average withdrawn (Section 4.3). Therefore, bank losses and closures must have been driven by the fear of losses and the ensuing bank runs and not direct failures themselves.



Figure 8: Average number of banks that fail in the financial system for equity and liquidity shocks of different sizes.

Note: Liquidity shocks are measured as a fraction of the liquid assets held by each bank, while equity shocks are a fraction of the loan positions of New York City banks. Shock sizes are varied between 0 and 1 in steps of 0.1, and the results are averaged over 100 different interbank networks for each point on the graph.

7 Conclusion

We use a new computational approach to study the interbank deposit network in the United States around the Panic of 1873. Our approach simultaneously estimates both the most likely network and the utility functions of banks at the time. We show that specie, as the ultimate form of liquidity, along with distance between counter-parties were key to banks' decision making processes. The networks resulting from this utility function provides a more accurate model for the withdrawal of funds after the 1873 shock than is possible using bank balance sheets and locations alone. The fit improves further if the model is used to reallocate funds across reserve cities after withdrawals are made. We also use the model to identify individual banks most likely to have panicked during the crisis from the resulting reallocation of interbank positions. Examination of exogenous data from those banks predicted to panic is consistent with the model's predictions. Finally we examine the resiliency of the financial network to direct contagion and find that only small numbers of banks fail in response to shocks - similar to that observed historically. Our results provide a clear separation between the relatively small direct effects of the crisis and the much larger informational or panic-based effects. As such the findings increase our understanding of these two interrelated concepts and opens a potential new research approach to understanding financial crises both past and future.

A Robustness

In this appendix we consider a number of additional sources of information and frictions which may further refine the network.

A.1 Interest on Deposits

One reason banks placed deposits in New York City banks as opposed to more local reserve agents was that the largest banks in New York paid interest on interbank deposits unlike those in any other city in the United States. The large New York banks were able to do this by lending the reserves on call cash to investors who could then purchase railroad bonds and other high-yield assets. The interest payments potentially create an additional attraction for non-New York City Banks to place their interbank deposits in New York.

This effect can be modeled by reducing the distance between the seven largest New York banks and all non-New York City banks in the system.²⁹ If distance is thought of as a physical space with an associated cost of travel, interest paid on deposits effectively reduces this transportation cost. Distance between banks and New York City banks are reduced by a fraction θ_I where θ_I is optimized along with δ and ν by the same process as described above. We find that the optimal $\theta_I = 0.01$ (to the nearest 0.005), meaning that interest payments do have an effect on banks' decision making. The effect, however, is relatively small - the overall fit between the network and observed data improves by approximately 1%. The remaining results are qualitatively similar as those presented above and are available upon request.

 $^{^{29}}$ We do not model the distance between the seven largest banks and other New York City banks as being reduced as other banks in the city would have had direct access to potential investors, and by convention we assume all banks within a given city have distance 0.

A.2 Redeeming Agents

The *Bankers Almanac* in the early 1870's provides a list of the 'New York Redeeming Agent' for each National Bank in the system. *Banker's Almanac* data are available for December 1872 and March 1874, and we manually digitized both lists and matched redeeming agents to banks in the Comptroller sample where appropriate. Despite being described as the New York Redeeming Agent, the banks listed were in many cases not in New York - there are many instances of banks in Boston, Philadelphia and Pittsburgh being named. Many name multiple banks as Redeeming Agents (the maximum being three). Several banks also name private banks as either their sole redeeming agents or one of a group. There are relatively few changes between the 1872 and 1874 lists with the exception of those banks who used Jay Cooke and Co. as Redeeming Agent in 1872.

The redeeming agents listed in the *Bankers Almanac* represent knowledge of existing relationships. They do not, however, necessarily mean that reserves are kept in these banks (see, for example, the cases where private banks are listed). The existing relationships, however, imply that these banks are closer in information space than their physical distance would imply. We therefore incorporate this knowledge by reducing the distance between every bank and its listed redeeming agents by some value θ_R . We optimize θ_R at the same time as δ and ν using the same process as described above and find that the optimal value is $\theta_R = 0.0$, i.e. distance is unchanged. This information therefore does not improve the fit of the model. This is because with a few exceptions the set of banks used as redeeming agents is relatively small, meaning that those banks would be overly favored by this scheme and so attract more deposits than are present on their balance sheets.

B Default algorithm

Section 6 of the paper analyzed the interbank network to understand how a shock to either the liquidity or equity of banks in New York City could spread throughout the financial system. To do this we used the fictitious default algorithm of Eisenberg and Noe (2001) to identify failing banks. This algorithm assumes limited liability in accounting for payments, i.e., that payments are limited by the banks' available cash balances and that creditor banks in the event of default are paid in proportion to the size of the debt owed and with priority of creditors over stock holders. We describe the algorithm below first defining the key terms (notation as in the original).

Let p_i be the vector of payments that bank *i* makes to each bank in the financial system and \bar{p}_i the total payment of *i* to all banks. $\bar{p} = (\bar{p}_1, \bar{p}_2, ..., \bar{p}_n)$ is then the 'total obligation vector,' which corresponds to the set of payments from every bank which would satisfy all creditors in the system.

Define Π to be the matrix with entries:

$$\Pi_{ij} \equiv \begin{cases} \frac{L_{ij}}{\bar{p}_i} & \text{if } \bar{p}_i > 0\\ 0 & \text{otherwise} \end{cases}$$
(13)

where L_{ij} is the liability of bank *i* to bank *j* as in the calculated interbank network. This matrix then represents the liability of each bank to each other bank as a fraction of the total debtor's liabilities.

The total cash flow for each bank is the sum of payments from other banks, described above, plus external payments, e_i , minus payments to other banks:

$$\sum_{j=1}^{n} \Pi_{ij} p_j + e_i - p_i \tag{14}$$

Let D(p) be the set of banks who default under payment vector p, then define

$$\Lambda(p)_{ij} = \begin{cases} 1 & \text{if } i = j \text{ and } i \in D(p) \\ 0 & \text{otherwise} \end{cases}$$
(15)

This matrix contains zeros everywhere except on the diagonal for those banks that do not default under payment vector p. The map $FF_{p'}$ is then defined as:

$$FF_{p'}(p) \equiv \Lambda(p'^T(\Lambda(p')p + (I - \Lambda(p')\bar{p})) + e) + (I - \Lambda(p'))(\bar{p})$$
(16)

which gives for those banks not in default under p', given by $\Lambda(p')$, the payment the bank

must make \bar{p} . For those banks in default, given by $I - \Lambda(p)$, it gives the bank's value. This assumes that defaulting banks pay p while non defaulting banks pay \bar{p} as set out above. Eisenberg and Noe (2001) showed that $F_{p'}$ has a unique fixed point f(p'). The fictitious default algorithm computes this iteratively starting from $p_0 = \bar{p}$ and proceeding through a series of fictitious defaults as $p^j = f(p^{j-1})$ until $p^j = p^{j-1}$, i.e. no more defaults occur and all payments can be met.

In our model the imposition of a shock could mean that the banks suffer a withdrawal of liquidity or impairments of assets. The logical interpretation of this would be negative external payments (e_i). In the above, however, external payments are required to be positive. We address this potential problem with the solution originally suggested in the same paper - by creating a fictitious additional bank which makes no payments but receives all payments due from the shock.

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