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Capital Income Taxation and Aggregate Instability

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March 2019

Abstract

This paper overturns the conventional wisdom that reliance on capital tax rate adjustment to ensure fiscal sustainability is immune to extrinsic uncertainty. The interaction of capital taxation and endogenous capital utilization generates *fiscal* increasing returns and factor share redistribution to induce sunspots expectations. Capital depreciation allowance debilitates this mechanism to preempt policy-induced instability while achieving budget objective. Self-fulfilling fluctuations can occur in real-world economies, unless their depreciation allowances are sufficiently higher or income tax rates lower than the current levels. This adds a short-run motivation to the long-run approach to capital taxation and the supply-side view of fiscal policy reforms.

JEL classification: E32; E62

Keywords: Capital income taxation; Depreciation allowance; Endogenous utilization; Fiscal increasing returns; Self-fulfilling prophecies

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1 Introduction

The implications of capital income taxation for efficiency or equality have long been studied in macroeconomics and held a center stage in public policy debate. While many advancements have been made in our understanding of the issue, no clear conclusion has been reached regarding whether capital income taxation should be avoided for reasons based on economic fundamentals. By contrast, in the literature that emphasizes the role of self-fulfilling prophecies, a consensus view has been held over the decades: A government that relies on adjusting capital (rather than labor) income tax rate to balance its budget or confine its indebtedness to ensure fiscal sustainability in financing its expenditures should be immune to extrinsic uncertainty and beliefs-driven instability.[1]

This conventional wisdom is overturned in the present paper that augments the neoclassical framework adopted in the existing literature with endogenous capital utilization, a real-world feature emphasized by Keynes (1936) and further developed by Taubman and Wilkinson (1970) among others. We show that, in this more realistic framework, reliance on capital income taxes to achieve budget objective may constitute a potential destabilizing force unrelated to economic fundamentals.[2] In particular, we analytically characterize the necessary and sufficient condition for multiplicity of equilibria, which may induce welfare-reducing extrinsic instability, by a large open interval for the long-term capital income tax rate, with the upper bound corresponding to the peak of the steady-state Laffer curve.

[1] This conventional view has been developed at least since Schmitt-Grohé and Uribe (1995, 1997). For related subsequent studies, see, Guo and Lansing (1998), Rocheteau (1999), Schmitt-Grohé and Uribe (2000), Guo and Harrison (2004), Gokan (2006, 2008), Giannisarou (2007), Stockman (2010), Anagnostopoulos and Giannitsarou (2013), Nourry et al. (2013), Ghilardi and Rossi (2014), and Huang et al. (2017, 2018), among many others.

[2] Like Schmitt-Grohé and Uribe (1995, 1997), our paper is related to the large strand of literature on multiple equilibria and sunspots. See, for example, Boldrin and Rustichini (1994), Benhabib and Farmer (1994, 1999), Wen (1998), Schmitt-Grohé (1997, 2000), Benhabib et al. (2000), Dupor (2001), Benhabib et al. (2001, 2002), Wang and Wen (2008), Jaimovich (2007, 2008), Huang and Meng (2012, 2014), Liu and Wang (2014), Lloyd-Braga et al. (2014), Dufourt et al. (2015), Bethune et al. (2018), and Benhabib et al. (2018).

Whereas this interval is wide open in our model with endogenous capital utilization, covering a broad range of capital income tax rates which may ever be thought of as empirically relevant, it degenerates to an empty set if capital utilization rate is assumed to be exogenous and constant as in the previous literature. Thus respecting the endogeneity of capital utilization turns the impossibility of indeterminacy on its head to render capital income taxation an important source of aggregate fluctuations driven by sunspots expectations.

Not only is this result regarding the destabilizing effects of capital taxation entirely new, but the mechanism behind it is also novel.[3] Starting from a standard neoclassical growth model with perfect competition and constant returns-to-scale production technology, we show that optimal choice in capital utilization rate can interact with the fiscal policy to generate both a factor share redistribution, from capital to labor, and a returns-to-scale effect, so that in equilibrium not only the effective degree of returns to scale of the aggregate production function becomes greater than unity, but in fact the effective output-labor elasticity itself goes above unity (which in the context of the model renders the labor demand schedule not only upward sloped but steeper than the labor supply schedule). The mechanism in this paper is therefore connected with the classic notion of "*fiscal* increasing returns to scale" emphasized by Blanchard and Summers (1987). It is through this mechanism can beliefs-driven instability be induced by fiscal policy that relies on capital income taxes to achieve budget objective.

Indeed, were capital utilization rate exogenously fixed, as in the previous literature in which the conventional wisdom is rooted, both the returns-to-scale and factor share redistribution effects would vanish, so the aggregate production function would remain at constant returns to scale while needless to say the output-labor elasticity would remain below unity in equilibrium, and extrinsic uncertainty would never emerge.[4] The intuition is as follows.

[3] The novelty of the mechanism in this paper can be appreciated in the context of the broad literature on multiple equilibria and sunspots (e.g., the studies referenced in Footnote 2). Please refer to Section 2.3 for more details.

[4] The essential ingredient of the concept of capital utilization as an optimal decision is that increasing capital utilization raises the user cost of capital via an acceleration of capital depreciation. As such, firms

Suppose agents contemplate lower capital tax rates. The corresponding higher investment would be coupled by greater labor input to produce more output, to which the tax base is proportional. Yet how much more output will be produced depends on how large the output-labor elasticity is, as capital input is pre-determined. With a lower than unitary output-labor elasticity, increases in output and so in the tax base would be insufficient to compensate for the reduction in the tax rate to generate the tax revenue needed for financing the government expenditure. This would invalidate the agents' initial contemplation so beliefs-driven fluctuations unrelated to economic fundamentals are preempted. By contrast, in our baseline setting with endogenous capital utilization rate, as the emergence of *fiscal* increasing returns to scale and factor share redistribution from capital to labor generates not only an increasing returns-to-scale aggregate production function but a greater than unitary effective output-labor elasticity, sufficiently more output will be produced and tax base generated with the contemplated increase in labor input to offset the reduction in the tax rate so as to satisfy the given fiscal policy. This renders the agents' initial expectations self-fulfilling thus the economy is susceptible to sunspots beliefs and endogenous fluctuations.

This brings us to another contribution of the paper. While our results stated above may issue a caution against reliance on capital income taxes to ensure fiscal sustainability, there are many reasons for why some form of capital taxation may be part of fiscal policy in the real world. For a government that does rely on capital income taxes to achieve its budget objective, we show, then it is important that it also gives tax allowances for capital depreciation expenses in order to preempt extrinsic instability in the face of optimal capital utilization. This is because, as we show, capital depreciation allowance debilitates *fiscal*

will not in general find it optimal to fully utilize the stock of capital, preferring to hoard some capital instead, so that they can use it more intensively when the return to doing so is relatively higher. Not only is this phenomenon much in line with the evidence documented in many empirical studies (see Chatterjee 2005 for a survey), but respecting this real-world feature has proven also important for deciphering a number of puzzles concerning growth and the business cycle (e.g., Greenwood et al. 1988, Basu 1996, Burnside and Eichenbaum 1996, Burnside et al. 1996, Wen 1998, Chatterjee 2005, Basu et al. 2006, and Huang et al. 2018).

increasing returns to scale and factor share redistribution brought about by the interaction of endogenous capital utilization and the fiscal policy, making sunspots expectations less likely to materialize while achieving budget objective. This second result of the paper regarding the stabilizing effects of capital depreciation allowances is entirely new as well. In particular, we show that a sufficiently high degree of capital depreciation allowances would reduce the effective degree of returns to scale of the aggregate production function all the way back to unity so sunspots equilibria would become entirely impossible regardless of the magnitude of capital income tax rates.

These results of capital income taxation and depreciation allowances on aggregate stability hold generally. As a showcase for their generality, we present similar results in a broader context that incorporates also public debt and labor income taxation. This showcase is useful not only because its setting is closer to the one observed in the real world, but for illustrating a third new point of this paper.[5] As has been argued in the previous studies, public debt whose sustainability concern is what usually motivates a balanced-budget debate can serve as an automatic stabilizer to exempt the fiscal policy practice from beliefs-driven instability.[6] Our showcase provides a counter example to this traditional view: Here we show that public debt can be destabilizing rather than stabilizing, although abstracting from one or more realistic features of our comprehensive setting can lead to the opposite conclusion. This is in contrast to the robustness of the stabilization role of capital depreciation allowances that generalizes to the more realistic settings.

To get a more practical feel, we apply the calibrated version of our comprehensive model to see its implications for the United States, the United Kingdom, and Japan. We find that extrinsic instability arises for capital depreciation allowances and income tax rates typically seen in these large economies, but that the economies can be stabilized if depreciation al-

[5] Our basic conclusions in this paper hold broadly, in an even more comprehensive setting with also consumption taxes included. These additional results are available upon request from the authors, but they are not presented in the paper in order to conserve space, and also because they do not add new insight other than providing additional robustness check of the main results.

[6] See, for example, Schmitt-Grohé and Uribe (1997) and Huang et al. (2018).

lowances are sufficiently higher or income tax rates lower than the current levels.[7] This adds a short-run motivation to the long-run approach to capital income taxation and the supply-side view of fiscal policy reforms.[8]

We organize the rest of this paper as follows. Section 2 sets up our baseline model with capital income taxation and shows how its interaction with endogenous capital utilization can generate *fiscal* increasing returns to scale and factor share redistribution from capital to labor to render the economy prone to sunspots beliefs and extrinsic uncertainty. Section 3 shows how capital depreciation allowances can debilitate such *fiscal* increasing returns to scale and factor share redistribution to help stabilize the economy. Section 4 generalizes these results to a more comprehensive setting that incorporates also public debt and labor income taxation. It is here we also provide a counter example to the conventional view that public debt can serve as a practically relevant stabilizer to exempt a balanced-budget fiscal policy from extrinsic uncertainty. We also discuss here the implications of our results for large economies like the United States, the United Kingdom, and Japan. Section 5 concludes. Most technical details are relegated to the appendix.

2 Capital taxation as a source of instability

This section shows how the interaction of a balanced-budget capital income tax rule and endogenous capital utilization can generate *fiscal* increasing returns to scale and factor share redistribution from capital to labor to render an otherwise standard constant returns-to-scale neoclassical economy prone to sunspots beliefs and extrinsic instability.

[7] It is worth noting that tax codes in countries around the world generally feature some degrees of capital depreciation allowances, and this feature of fiscal policy has been studied before for its various implications. See, among others, Sinn (1987), Guo and Lansing (1999), Strulik and Trimborn (2012), Mendoza et al. (2014), and D'Erasmo et al. (2017). The present paper is the first to show the stabilization role of this feature of fiscal policy in a context where the government relies on capital taxation to achieve budget objective in the face of optimal capital utilization.

[8] For the long-run approach to capital income taxation, see, for example, Judd (1985) and Chamley (1986).

2.1 Basic setting

Given initial capital stock k_0, the representative household chooses paths for consumption, c_t, labor, l_t, investment, i_t, capital stock, k_t, for $t > 0$, and capital utilization rate, u_t, to maximize the present discounted value of its lifetime utility

$$\int_0^\infty (\log c_t - \varphi l_t)\, e^{-\rho t} dt, \tag{1}$$

for marginal dis-utility from working $\varphi > 0$,[9] and a subjective discount rate $\rho \in (0, 1)$, subject to

$$c_t + i_t = w_t l_t + r_t (u_t k_t) - T_t, \tag{2}$$

$$\dot{k}_t = i_t - \delta_t k_t, \tag{3}$$

$$T_t = \tau_t^k r_t (u_t k_t), \tag{4}$$

where τ_t^k denotes the capital income tax rate and T_t government tax revenue, w_t the wage rate, r_t the pre-tax rental rate for capital services, and $\delta_t = \delta(u_t)$ the rate of capital depreciation, which is a function of the utilization rate of capital, with $\delta'(u_t) > 0$ and $\delta''(u_t) \geq 0$. A parametrization that satisfies these properties takes the form, $\delta(u_t) = \tilde{\theta} u_t^\theta / \theta$, with $\theta > 1$ and $\tilde{\theta} > 0$ so that $\delta(u_t) \in [0, 1]$.[10] Denoting by λ_t the marginal utility of income, the first-order

[9] The linearity of the period utility function in labor services is a consequence of aggregation when labor is assumed to be indivisible and such a utility function is consistent with any labor supply elasticity at the individual level (e.g., Hansen 1985, Rogerson 1988). Such a formulation is commonly adopted in the RBC-based indeterminacy literature, including Schmitt-Grohé and Uribe (1997). See Benhabib and Farmer (1999) for a survey. Furthermore, in a model with nominal wage rigidity, Huang and Meng (2012) find that indeterminacy arises for plausible increasing returns regardless of the magnitude of the labor supply elasticity.

[10] This approach in modeling the depreciation rate of capital as an increasing convex function of capital utilization rate is similar to that in Greenwood et al. (1988), and Jaimovich and Rebelo (2009) for a centralized system, but is more closely related to that in Greenwood and Huffman (1991), Finn (1995, 2000), and D'Erasmo et al. (2017) for a decentralized economy. It generalizes Keynes's notion of the user cost of capital – higher utilization causes faster depreciation, at an increasing rate, because of wear and tear on the capital stock. Note that here $\tilde{\theta} > 0$ is a scaling parameter that is adjusted to ensure that the steady state

conditions associated with the household's problem are

$$c_t^{-1} = \lambda_t, \tag{5}$$

$$\varphi = \lambda_t w_t, \tag{6}$$

$$\delta'(u_t)\, k_t = r_t \left(1 - \tau_t^k\right) k_t, \tag{7}$$

$$\dot{\lambda}_t = \lambda_t \left[\rho + \delta_t - \left(1 - \tau_t^k\right) r_t u_t\right]. \tag{8}$$

The representative firm hires effective capital services, $u_t k_t$, and labor, l_t, to maximize its profit, $y_t - r_t(u_t k_t) - w_t l_t$, taking the factor rental rates as given. The production function is the standard Cobb-Douglas with constant returns to scale,

$$y_t = (u_t k_t)^{\alpha}\, l_t^{\beta},\ \alpha + \beta = 1,\ \alpha \in (0, 1). \tag{9}$$

Perfect competition in factor and product markets implies the firm's optimality conditions,

$$r_t = \alpha \frac{y_t}{u_t k_t},\ \ w_t = \beta \frac{y_t}{l_t}. \tag{10}$$

The government chooses the capital income tax rate τ_t^k to run a balanced budget,

$$G = T_t = \tau_t^k r_t (u_t k_t), \tag{11}$$

with a pre-set government expenditure G.

Finally, the goods market-clearing condition for the economy is

$$c_t + G + \dot{k}_t + \delta_t k_t = y_t. \tag{12}$$

2.2 Local dynamics and instability

In what follows, a variable with no time index denotes its steady-state value. With some algebra, we derive the following steady-state equilibrium relations: $y/k = (\rho + \delta) / \left[\alpha\left(1 - \tau^k\right)\right]$, $\varphi l = (1 - \alpha)(\rho + \delta) / \left[\left(1 - \alpha\tau^k\right)\rho + (1 - \alpha)\delta\right]$, and $c/y = 1 - \alpha\left(\rho\tau^k + \delta\right) / (\rho + \delta)$. We then

with endogenous capital utilization is the same as the steady state with constant capital utilization under a given fiscal policy.

show that, for a given steady-state capital income tax rate τ^k, there exists a unique steady state for the other variables. We can also derive a steady-state Laffer curve relationship between the capital income tax rate and the tax revenue with

$$\frac{dT}{d\tau^k} = \frac{T}{\tau^k} \frac{\rho\Omega}{(1-\alpha)(1-\tau^k)[\rho(1-\alpha\tau^k)+\delta(1-\alpha)]},$$

where Ω when viewed as a function of τ^k is convex given by

$$\Omega\left(\tau^k\right) \equiv \alpha^2 \left(\tau^k\right)^2 - [1+(1-\alpha)\delta/\rho]\,\tau^k + (1-\alpha)[1+(1-\alpha)\delta/\rho]\,. \tag{13}$$

Let $s \equiv [\rho+(1-\alpha)\delta]/(\rho+\delta) \in (1-\alpha, 1)$. It can be verified that $\Omega(1-\alpha) > 0$ and $\Omega(s) < 0$. We can then show that there is a unique long-run capital income tax rate $\bar{\tau} \in (1-\alpha, s)$ that reaches the peak of the steady-state Laffer curve.[11]

Linearizing the equilibrium dynamics around the steady state, we show that local (in)stability property of this neoclassical economy can be analyzed by examining the following system of two first-order linear differential equations in k_t and λ_t,

$$\begin{pmatrix} \dot{k}_t \\ \dot{\lambda}_t \end{pmatrix} = \mathbf{J} \begin{pmatrix} k_t - k \\ \lambda_t - \lambda \end{pmatrix}, \tag{14}$$

where the trace and determinant of the two by two Jacobian matrix $\mathbf{J}$ are given by

$$\mathcal{T} = -\frac{\Delta}{\tau^k - \underline{\tau}},\ \mathcal{D} = \frac{\Lambda}{\tau^k - \underline{\tau}}\Omega, \tag{15}$$

respectively, where $\underline{\tau} \equiv \rho/(\rho+\delta)$, $\Delta \equiv \rho^2/(\rho+\delta)$, and $\Lambda \equiv (\rho/\alpha)^2/\left(1-\tau^k\right)$. Since (14) contains one predetermined variable (k_t) and one jump variable (λ_t), the dynamic system exhibits instability if and only if the two eigenvalues of $\mathbf{J}$ are both negative (i.e., if and only if $\mathcal{T} < 0$ and $\mathcal{D} > 0$), while it is saddle-path stable if and only if the two eigenvalues are of opposite signs. The system has no equilibrium solutions that converge to the steady state if and only if the two eigenvalues are both positive. Noting that $\Omega\left(\tau^k\right) > (<)\ 0$ for $\tau^k < (>)\ \bar{\tau}$, we obtain the first main result of this paper.

[11] As $\Omega(1) < 0$ and $\Omega(+\infty) > 0$, the other root to $\Omega\left(\tau^k\right) = 0$ is greater than 1 and so is irrelevant for our analysis.

Proposition 1. *The fiscal policy with capital income taxation (11) induces indeterminacy of equilibrium if and only if*

$$\underline{\tau} < \tau^k < \bar{\tau}. \tag{16}$$

Since $\bar{\tau} > 1 - \alpha$, it is sufficient to have $\underline{\tau} < 1 - \alpha$, or, equivalently, $\alpha < \delta/(\rho + \delta)$, in order for the open interval in (16) to be nonempty. This is easily the case for all empirically reasonable calibrations of the deep parameters, under which $(\underline{\tau}, \bar{\tau})$ remains robustly a large open interval covering a broad range of capital income tax rates which may ever be thought of as practically relevant. To get a more concrete feel, we set $\rho = 0.04$, $\delta = 0.1$, and $\alpha = 0.3$, as in Schmitt-Grohé and Uribe (1997). Then we have $(\underline{\tau}, \bar{\tau}) = (0.286, 0.717)$. The capital income tax rates in the US, the UK, and Japan all fall into this wide range. Hence reliance on capital income taxes to ensure fiscal sustainability can make these countries prone to extrinsic uncertainty and beliefs-driven instability, subjecting their economies to persistent and recurrent fluctuations in aggregate activities even in the absence of shocks to their fundamentals.[12]

One way to preempt this problem is to ensure a low enough long-term capital income tax rate. We can show, when confronted by this clear and present threat (i.e., when $(\underline{\tau}, \bar{\tau}) \neq \emptyset$), saddle-path stability of the dynamic system is ensured if and only if $\tau^k < \underline{\tau}$ or $\tau^k > \bar{\tau}$ (as in either case $\mathcal{D} < 0$). This is to say that, in light of the above calibration, if the long-term capital income tax rate is brought below 0.286, then the economy would for sure be saddle-path stable. This adds a short-run motivation to the long-run approach to capital income taxation as well as to the supply-side view of the fiscal policy reforms advocating for lower capital income tax rates.[13]

[12] We can show, similarly as in Shigoka (1994), that local indeterminacy of the perfect-foresight equilibrium implies the existence of stationary sunspots equilibria.

[13] Theoretically saddle-path stability of the economy can also be ensured with a sufficiently high long-term capital income tax rate - greater than 0.717 in the context of the above calibration. This theoretical result has little empirical relevance. In addition, we can show that very large steady-state tax rates can distort the economy to such a degree that in equilibrium the aggregate production function becomes a decreasing function of the capital and labor inputs.

Figure 1 provides a graphic illustration of the above results.

2.3 Inspecting the mechanism

Our result regarding the destabilizing effects of capital taxation is entirely new. The mechanism behind it is also novel. Our basic setting is a standard neoclassical growth model with perfect competition and constant returns-to-scale production function. Yet, as we will show below, optimal choice in capital utilization rate can interact with the fiscal policy to generate both a factor share redistribution, from capital to labor, and a returns-to-scale effect, so that in equilibrium not only the effective degree of returns to scale of the aggregate production function becomes greater than unity, but in fact the effective output-labor elasticity itself goes above unity (which in the context of the model renders the labor demand schedule not only upward sloped but steeper than the labor supply schedule). This is a totally new type of increasing returns in the context of the broad literature on indeterminacy and sunspots, which we have referred to as *fiscal* increasing returns to scale à là Blanchard and Summers (1987). It is through this mechanism can beliefs-driven instability be induced by fiscal policy that relies on capital income taxes to achieve budget objective.

To help highlight the mechanism in a transparent way, it is useful to denote by $\hat{x}_t = \log x_t - \log x$ the percentage deviation of a variable from its steady state. Using the equilibrium conditions (7), (9), (10), and (11) to substitute out the optimal capital utilization rate, we obtain a log-linear representation of the effective aggregate production function,

$$\widehat{y}_t = \widetilde{\alpha}\widehat{k}_t + \widetilde{\beta}\widehat{l}_t = \left(1 + \eta_f\right)\left[\eta_k \alpha \widehat{k}_t + \eta_l \beta \widehat{l}_t\right], \tag{17}$$

where the effective output-capital elasticity, $\widetilde{\alpha}$, the effective output-labor elasticity, $\widetilde{\beta}$, along with the other three parameters introduced in (17), namely, η_f, η_k, and η_l, are given by

$$\widetilde{\alpha} = \left(1 + \eta_f\right)\eta_k \alpha, \quad \widetilde{\beta} = \left(1 + \eta_f\right)\eta_l \beta;$$

$$\eta_f = \frac{\alpha}{\theta}\frac{\tau^k}{s - \tau^k}, \quad \eta_k = \frac{\theta - 1}{\theta - \alpha}, \quad \eta_l = \frac{\theta}{\theta - \alpha}.$$

Some important observations follow.

First, $\eta_k < 1$, $\eta_l > 1$, and $\eta_k\alpha + \eta_l\beta = 1$, for $\theta \in (1, \infty)$. This shows a factor share redistribution, from capital to labor. As can be verified, this redistribution effect is stronger, the greater is the elasticity of capital utilization rate with respect to capital depreciation rate, which is measured by the inverse of θ. In the limiting case with $\theta = \infty$, we have $\eta_k = \eta_l = 1$ and this redistribution effect vanishes. This limiting case is interesting since as we can show it coincides with the case of a constant capital utilization rate.

Second, $\eta_f > 0$, for $\theta \in (1, \infty)$ and $\tau^k \in (0, s)$, while recalling that s is strictly greater than the upper bound $\bar{\tau}$ for long-term capital income tax rate that is relevant to the indeterminacy issue. The effective degree of *fiscal* increasing returns to scale is measured by the magnitude of η_f, which is larger, the larger is the elasticity of capital utilization rate to capital depreciation rate $1/\theta$, or\and the greater is the long-term capital income tax rate τ^k. Further, if τ^k is greater than the lower bound $\underline{\tau}$ for long-term capital income tax rate that is relevant to the problem of concern, then the effective output-labor elasticity itself becomes greater than unity, as we can verify that $\widetilde{\beta} = 1 + \alpha(\tau^k - \underline{\tau})/\left(s - \tau^k\right)$ is greater than unity and increases with τ^k for $\tau^k \in (\underline{\tau}, s)$. If $\tau^k = 0$, then $\eta_f = 0$ though $\widetilde{\beta} = \eta_l\beta > \beta$, so the returns-to-scale effect vanishes though the redistribution effect stays. In the limiting case with $\theta = \infty$, we have not only $\eta_f = 0$ but $\widetilde{\beta} = \beta$ (and $\widetilde{\alpha} = \alpha$), and therefore, both the returns-to-scale effect and the redistribution effect disappear. As stated above, we can show that this limiting case coincides with the case of a constant capital utilization rate.

This observation leads to the following immediate corollary of Proposition 1.

Corollary 1. *Indeterminacy of equilibrium under the fiscal policy with capital income taxation (11) requires that not only the aggregate production function be effectively increasing returns to scale with respect to capital and labor but the effective output-labor elasticity itself be greater than unity.*

The above two observations make it clear that it is the joint presence of endogenous capital utilization and reliance on capital income taxation to achieve the government's budget objective that generates the *fiscal* increasing returns to scale and factor share redistribution in our otherwise standard neoclassical setting. In particular, as just illustrated above, if capital utilization rate were exogenously fixed, both effects would vanish, and the effective aggregate production function would remain to be constant returns to scale while the output-labor elasticity would remain below unity so as a result extrinsic uncertainty would never arise.

To drive our illustration home, we present below the solution to optimal capital utilization rate,

$$\widehat{u}_t = e_1 \widehat{l}_t - e_2 \widehat{k}_t, \text{ where } e_1 = \frac{1-\alpha}{\theta(s-\tau^k)}, \; e_2 = \frac{1-\alpha-\tau^k}{\theta(s-\tau^k)}. \tag{18}$$

The fact that $e_1 > e_2$ and the two elasticities of optimal capacity utilization rate with respect to labor and capital, e_1 and $-e_2$, tend to have opposite signs highlights the returns-to-scale and factor share redistribution effects, as optimal decision rule prescribes that capital be used less intensively with its existing stock but more intensively with labor whereby the intensity of utilization respond more substantially to the labor input than to the existing capital stock. Clearly, in the case with a constant capital utilization rate (which coincides with the limiting case that $\theta = \infty$), $e_1 - e_2 = 0$, so both the returns-to-scale and factor share redistribution effects disappear. In our setting with endogenous capital utilization in the face of capital income taxation, $e_1 - e_2 = \eta_f / \alpha > 0$, which illustrates the root cause of the *fiscal* increasing returns to scale and factor share redistribution from capital to labor in our model, the mechanism for generating the paper's first main result.

The novelty of this paper's mechanism can be best appreciated in the context of the broader literature on multiplicity of equilibria and sunspots. In the classic literature originated by the seminal works of Benhabib and Farmer (1994) and Farmer and Guo (1994), the preassumption of an increasing returns-to-scale aggregate production function is a necessary starting point in order to have any hope to generate indeterminacy of equilibrium; as a matter of fact, the required degree of increasing returns to scale for indeterminacy is typically too large to be empirically justifiable. By incorporating variable capital utilization into this classic framework, Wen (1998) lowers the required degree of increasing returns to scale for indeterminacy to a smaller and empirically more plausible level; nevertheless, the preassumption of an increasing returns-to-scale aggregate production function remains a necessary starting point - if the aggregate production function is assume to be of constant returns to scale to begin with, it will remain at constant returns to scale in equilibrium even with endogenous capital utilization and indeterminacy will remain entirely impossible.

On the other side, the literature pioneered by Schmitt-Grohé and Uribe (1997) discovers a policy-induced mechanism for indeterminacy that does not require any increasing returns to scale. A common assumption here is constant returns to scale coupled with constant capital utilization, where a consensus reached is that reliance on labor income taxation to achieve budget objective induces aggregate instability unrelated to economic fundamentals, but reliance on capital income taxation to do so is immune to extrinsic uncertainty. In the context where a government does rely on labor income taxation to achieve budget objective, Huang et al. (2018) show that incorporating endogenous capital utilization does *not* alter the fact that aggregate production function remains to be constant returns to scale even though it does increase the likelihood of indeterminacy through creating a factor share redistribution from capital to labor. This paper is the first to show that, if the government relies on capital income taxation to achieve budget objective, then incorporating endogenous capital utilization turns the aggregate production function from constant into increasing returns to scale while generates a factor share redistribution from capital to labor, which together render the economy prone to sunspots expectations and extrinsic instability. This mechanism is totally new and provides a policy-based micro-foundation to the existence of aggregate increasing returns to scale with respect to capital and labor inputs. It is through this mechanism can extrinsic uncertainty be induced by fiscal policy that relies on capital taxation to achieve budget objective.

3 Depreciation allowances as a stabilization device

While the result in Section 2 may issue a caution against reliance on capital income taxes to ensure fiscal sustainability, there are many reasons for why some form of capital taxation may be part of fiscal policy in the real world. This takes us to the second contribution of the present paper. This section is devoted to showing that, if a government does rely on capital taxation to achieve budget objective, then it is important that it also gives tax allowances for capital depreciation expenses in order to preempt extrinsic instability in the face of optimal capital utilization. This is because, as we show below, depreciation allowance debilitates the *fiscal* increasing returns to scale and factor share redistribution brought about by the interaction of endogenous capital utilization and the fiscal policy, making sunspots expectations less likely to materialize whereby achieving budget objective. In particular, we will show that with a sufficiently high degree of depreciation allowance sunspots equilibria can occur only if the long-term capital income tax rate is implausibly large, and that a full degree of depreciation allowance would reduce the effective degree of returns to scale of the aggregate production function all the way back to unity so sunspots equilibria would become entirely impossible regardless of the magnitude of capital income tax rates.

Denote by $\mu \in [0, 1]$ the degree of tax allowance for capital depreciation. The fiscal policy (11) in Section 2 is then modified as

$$G = T_t = \tau_t^k [r_t(u_t k_t) - \mu \delta_t k_t], \tag{19}$$

which encompasses Section 2 as a special case, the case with zero depreciation allowance or $\mu = 0$.

We show in the appendix that there exists a unique root $\overline{\tau}^D$ lying strictly between 0 and 1 that solves the following equation in τ^k,

$$\frac{\rho + (1-\mu)\,\delta}{(1-\tau^k)\,\rho} + \frac{\alpha}{1-\alpha}\frac{\rho + (1-\mu)\,\delta}{\rho + (1-\mu\tau^k)\,\delta} + \frac{\delta\,(\alpha-\mu)}{\alpha\,(\rho+\delta-\mu\delta)}\left(\frac{1-\mu}{1-\mu\tau^k} - 1\right) - \frac{\rho + (1-\alpha)\,\delta}{\alpha\rho\tau^k} = 0, \tag{20}$$

where for later reference we will denote the left side of (20) by $\Omega^D\left(\tau^k\right)$ viewed as a function of τ^k.

Let $\omega \equiv [(1-\mu)\,\delta]^2 / \rho / [\rho + (1-\mu)\,\delta] \geq 0$. In the appendix we also show that the smaller root of the following quadratic equation in τ^k, denoted as $\underline{\tau}^D$, lies in $(0, (1+\omega)^{-1}]$,

$$(1+\omega)\,\mu\left(\tau^k\right)^2 - \left(1 + \mu + \frac{\rho+\delta}{\delta}\omega\right)\tau^k + 1 = 0, \tag{21}$$

and that the left side of (21) as a convex function of τ^k, denoted as $\Pi^D\left(\tau^k\right)$, is strictly decreasing with τ^k, for $\tau^k \in \left[0, \underline{\tau}^D\right]$.

The following proposition shows that the indeterminacy region for this more general setting continues to be characterized by an open interval for the long-term capital income tax rate.

Proposition 2. *Indeterminacy of equilibrium can occur under the fiscal policy with capital income taxation and depreciation allowance (19) if and only if*

$$\underline{\tau}^D < \tau^k < \overline{\tau}^D. \tag{22}$$

With some algebra we can verify that, for the case with zero capital depreciation allowance, the upper and lower bounds in (22) degenerate to the upper and lower bounds in (16); that is, for $\mu = 0$ we have $\overline{\tau}^D = \overline{\tau}$ and $\underline{\tau}^D = \underline{\tau}$ (note that the latter can be easily verified by solving now the linear equation (21) for the unique root $\rho/(\rho+\delta)$). Figure 2 which is plotted under the calibrated parameter values illustrates how the indeterminacy region in (22) generally shrinks as μ increases and eventually becomes an empty set with a high enough μ. We can in fact *analytically* prove that $\underline{\tau}^D$ strictly increases in μ, and that if $\mu = 1$ then the quadratic equation (21) has a repeated unit root $\underline{\tau}^D = 1$ so the open interval characterized by (22) is indeed empty and multiplicity of equilibria would become entirely impossible regardless of the value of τ^k.

The corollary below summarizes these *analytical* results.

Corollary 2. *The lower bound on the long-term capital income tax rate required for indeterminacy of equilibrium presented in Proposition 2 is a strictly increasing function of the degree of capital depreciation allowance; and, with full allowance, it is equal to* 1 *so indeterminacy can never occur regardless of the magnitude of the capital income tax rate.*

The following corollary of Proposition 2, which generalizes Corollary 1, serves as a starting point for understanding the intuition behind the stabilization role of capital depreciation allowances.

Corollary 3. *Indeterminacy of equilibrium under the fiscal policy with capital income taxation and depreciation allowance (19) requires that not only the aggregate production function be effectively increasing returns to scale with respect to capital and labor but the effective output-labor elasticity itself be greater than unity.*

We can now demonstrate that capital depreciation allowance helps stabilize the economy because it debilitates the *fiscal* increasing returns to scale and factor share redistribution from capital to labor brought about by the interaction of endogenous capital utilization and the fiscal policy. To show this in a more transparent way, we apply a similar approach as in Section 2.3 to obtain a log-linear representation of the effective aggregate production function for this general case,

$$\widehat{y}_t = \widetilde{\alpha}^D \widehat{k}_t + \widetilde{\beta}^D \widehat{l}_t = \left(1 + \eta_f^D\right) \left[\eta_k^D \alpha \widehat{k}_t + \eta_l^D \beta \widehat{l}_t\right], \tag{23}$$

$$\widetilde{\alpha}^D = \left(1 + \eta_f^D\right) \eta_k^D \alpha, \quad \widetilde{\beta}^D = \left(1 + \eta_f^D\right) \eta_l^D \beta;$$

$$\eta_f^D = \frac{\alpha\delta}{\rho + \delta} \frac{\tau^k}{s^D \left(1 - \tau^k\right) - (1 - s) \tau^k}, \quad \eta_k^D = \frac{\theta^D - 1}{\theta^D - \alpha}, \quad \eta_l^D = \frac{\theta^D}{\theta^D - \alpha},$$

where $\theta^D \equiv 1 + \rho / \left[\left(1 - \tau^k \mu\right) \delta\right]$ and $s^D \equiv \left[\frac{\rho/\delta + (1-\alpha)\left(1-\tau^k\right)}{1-\mu} + \frac{\theta^D - (1+\rho/\delta) + (1+\rho/\delta)(1-\alpha)\tau^k}{1+\rho/\delta-\mu}\right] \frac{\delta}{\rho+\delta}$.

It can be verified that, if $\mu = 0$, then $\theta^D = 1 + \rho/\delta$ and $s^D = s$, so (23) degenerates to (17) with $\eta_f^D = \eta_f$, $\eta_k^D = \eta_k$, and $\eta_l^D = \eta_l$, and that $\partial\theta^D/\partial\mu > 0$ for $\mu \in [0, 1]$ and $\partial s^D/\partial\mu > 0$ for $\mu \in [0, 1)$ whereby $s^D \to +\infty$ as $\mu \to 1^-$.

Using the above fact we can accomplish our demonstration as follows.

First, for $\mu \in [0, 1]$, although it remains true that $\eta_k^D < 1$ and $\eta_l^D > 1$ with $\alpha\eta_k^D + (1 - \alpha)\eta_l^D = 1$, we can prove that $\partial\eta_k^D/\partial\mu > 0$ and $\partial\eta_l^D/\partial\mu < 0$. This is to say that, even though the interaction of endogenous capital utilization and the fiscal policy in this general case continues to create factor share redistribution from capital to labor, the degree of such factor share redistribution is smaller, the greater is the degree of capital depreciation allowance.

Second, for $\mu \in [0, 1)$ and $\tau^k \in \left(0, s^D/(1 + s^D - s)\right)$[14], even though it remains true that

[14] Outside this range of long-term capital income tax rates indeterminacy of equilibrium would never become an issue in the first place. See the proof of Corollary 3 in the appendix.

$\eta_f^D > 0$, we can prove that $\partial\eta_f^D/\partial\mu < 0$ and $\eta_f^D \to 0^+$ as $\mu \to 1^-$. In other words, although the interaction of endogenous capital utilization and the fiscal policy in this general case continues to create *fiscal* increasing returns to scale, the degree of such *fiscal* increasing returns to scale is smaller, the greater is the degree of capital depreciation allowances. And, with a full degree of depreciation allowances, such returns-to-scale effect would disappear entirely so the effective degree of returns to scale of the aggregate production function would go all the way back to unity and indeterminacy of equilibrium would become entirely impossible regardless of the magnitude of τ^k.

In actuality, as Figure 2 illustrates, some partial degree of capital depreciation allowances would be sufficient to preempt sunspots equilibria irrespective of the long-term capital income tax rate.

4 Taking into account public debt and labor income tax

We have shown that capital depreciation allowances can serve as a stabilization device to help preempt extrinsic uncertainty arising from reliance on capital income taxation to ensure fiscal sustainability in the face of optimal capital utilization. These results regarding capital taxation as a source of beliefs-driven instability and the stabilization role of capital depreciation allowances in helping resolve the problem hold quite broadly and generalize to more realistic settings that are closer to the real world than our benchmark model.

Our analysis in this section takes four steps progressively.

4.1 Taking into account public debt

In the first step, we enrich the model in Section 2 with public debt, a real world feature whose sustainability concern is what typically motivates a balanced-budget debate in the first place and which is also commonly believed to be an automatic stabilizer that can exempt such a fiscal policy practice from extrinsic instability. With public debt taken into consideration, the household budget constraint (2), government revenue (4), and fiscal policy (11) are amended respectively as,

$$c_t + i_t = w_t l_t + r_t \left(u_t k_t\right) + R_t B - T_t, \tag{24}$$

$$T_t = \tau_t^k r_t(u_t k_t) + \tau_t^k R_t B, \tag{25}$$

$$G + \left(1 - \tau_t^k\right) R_t B = \tau_t^k r_t(u_t k_t), \tag{26}$$

where B denotes the pre-existing stock of public debt which is held as a constant by virtue of the balanced-budget rule, and R_t denotes the rate of interest paid on the debt, which in equilibrium must equal $\left[r_t u_t - \delta\left(u_t\right) / \left(1 - \tau_t^k\right)\right]$ as implied by a no-arbitrage condition. The other equilibrium conditions are accordingly generalized from those in Section 2.

We can analytically prove that the indeterminacy region in this extended model continues to be characterized by an open interval for the long-term capital income tax rate, which does shrink with the steady-state public debt-GDP ratio, $s_b \equiv B/y$, consistent with the conventional wisdom, but which remains robustly wide open, covering a significant range of the long-term capital income tax rates, for all empirically conceivable values of s_b.[15] Figure 3 which is plotted under the calibrated parameter values demonstrates this general result. In other words, even in the presence of public debt, capital income taxation continues to be a practically relevant source of extrinsic instability.

4.2 Adding on top labor income tax

In the second step, we add on top of the setting in Section 4.1 labor income tax so the model is closer further to the real world. With a fixed labor income tax rate τ^l taken into consideration,[16] the government revenue (25) and fiscal policy (26) are amended respectively as,

$$T_t = \tau_t^k r_t(u_t k_t) + \tau_t^k R_t B + \tau^l w_t l_t, \tag{27}$$

[15] These analytical results are not presented here due to the space constraint, but they are available upon request from the authors.

[16] For the case with endogenous factor income tax rates, please refer to Huang et al. (2018).

$$G + \left(1 - \tau_t^k\right) R_t B = \tau_t^k r_t(u_t k_t) + \tau^l w_t l_t, \tag{28}$$

where the first-order condition (6) changes to,

$$\varphi l_t = (1 - \alpha)\left(1 - \tau^l\right) \lambda_t y_t. \tag{29}$$

The other equilibrium conditions are accordingly generalized from those in Section 4.1.

We can analytically prove that the indeterminacy region in this further-closer-to-reality model continues to be characterized by an open interval for the long-term capital income tax rate, which is not only wide open, covering a significant range of the long-term capital income tax rates, but can in fact expand with the steady-state public debt-GDP ratio, s_b.[17] In other words, not only does reliance on capital income taxation to achieve budget objective continue to be a practically relevant source of extrinsic instability, but the presence of public debt actually exacerbates the situation by increasing the likelihood of sunspots uncertainty. This is to say that, public debt can be destabilizing rather than stabilizing in this more realistic environment, overturning completely the conventional wisdom. Figure 4 illustrates this striking result, which is plotted under the calibrated parameter values, with the labor income tax rate taken to be the average (0.2653) over those reported in Table 1 for the three large economies to be examined later (i.e., US, UK, and Japan).

Some intuition behind the role reversal of public debt

The analyses in the two steps above together demonstrate not only the general destabilization effect of reliance on capital tax rate adjustment to ensure fiscal sustainability, but also the role reversal of public debt in this regard. Why does public debt switch from being a stabilizer in Section 4.1 to being a de-stabilizer in Section 4.2 when the setting becomes further more realistic? We provide here some intuition for this role reversal of public debt.

Suppose agents contemplate a lower capital income tax rate. Then greater capital service would be coupled by larger labor input to produce greater output and factor incomes. Higher capital income would then call capital tax rate into being lowered indeed in order to maintain

[17] These analytical results are not presented here in order to conserve space, but they are available upon request from the authors.

a balanced government budget. This is the case in Section 2. In Section 4.1 that takes into account public debt, greater debt payment takes place (due to higher interest rate that goes up with output and the rate of return on capital) to balance the higher capital income, thus government budget could be re-balanced without requiring capital tax rate being lowered. This is why in Section 4.1 public debt helps invalidate the agents' initial contemplation to stabilize the economy. In Section 4.2 that also takes into account labor income tax, however, the tax base is much broader and tax income much larger (given the 70% share of labor income in value-added), requiring a bigger-than-contemplated reduction in capital tax rate to maintain a balanced government budget. But, now, the greater debt payment offsets some of the larger tax base and income, so the contemplated reduction in capital tax rate could re-balance government budget. This is why in Section 4.2 public debt helps validate the agents' initial contemplation to destabilize the economy.

4.3 Depreciation allowances as a robust stabilization device

The analyses in the previous steps also suggest a need to check the robustness of the stabilization role of capital depreciation allowances demonstrated in Section 3. To this end, we further enrich the setting in Section 4.2 with capital depreciation allowances. In what follows, we will use the same notation $\mu \in [0, 1]$ as in Section 3 to denote the degree of tax allowances for capital depreciation. The government revenue and fiscal policy are now given by,

$$T_t = \tau_t^k \left(r_t u_t - \mu \delta_t \right) k_t + \tau_t^k R_t B + \tau^l w_t l_t, \tag{30}$$

$$G + \left(1 - \tau_t^k\right) R_t B = \tau_t^k \left(r_t u_t - \mu \delta_t \right) k_t + \tau^l w_t l_t, \tag{31}$$

where B denotes the pre-existing stock of public debt and R_t denotes the rate of interest paid on the debt, as in Section 4.1. No-arbitrage condition now implies that $R_t = r_t u_t - \delta(u_t)(1 - \mu\tau_t^k)/(1 - \tau_t^k)$. The other equilibrium conditions are accordingly generalized from those in Section 4.2.

Transparent analytical results are hard to obtain in this most comprehensive setting, so in this third step we appeal to numerical analysis. We find that capital depreciation allowance remains a robustly powerful stabilization device in this most realistic setting where it continues to help insulate the economy against extrinsic instability.

This is illustrated by Figure 5 which is plotted under the calibrated parameter values, with the labor income tax rate taken to be the average (0.2653) over those reported in Table 1 for the three large economies to be examined below (i.e., US, UK, and Japan). The upper panel of the figure is plotted with the steady-state public debt-GDP ratio s_b taken to be also the average (0.9753) over those reported in Table 1 for the three large economies, whereas the lower panel is plotted with s_b taken to be the value for Japan, equal to 1.429, highest among the three large economies. This latter case is interesting not only for the sake of Japan itself, but also because it is indicative of an economy like the US going forward.[18]

Two take-home messages emanate from Figure 5.

First, contrasting the two panels in the figure makes it clear that a rising public debt-GDP ratio can quickly expand the indeterminacy region, more dramatically so with lower capital depreciation allowance. This is a quantitative illustration in this most realistic setting of the destabilization role of public debt shown in Section 4.2 where the degree of capital depreciation allowance is set to zero.

Second, capital depreciation allowance shrinks the indeterminacy region, more rapidly so with a higher public debt to GDP ratio. In other words, the role of capital depreciation allowance as a stabilization device is not only robustly general, but more powerful in a situation where a higher public debt to GDP ratio renders the economy more fragile to beliefs-driven fluctuations.

In sum, in this more realistic economic setting, especially in the face of rising public debt that may bring with it increasing extrinsic uncertainty, capital depreciation allowances

[18] The high and rapidly rising public debt is a pressing national issue in the US. According to the Long-Term Budget Outlook conducted by the US Congressional Budget Office (CBO), the US public debt-GDP ratio may quickly approach a level comparable to that reported in Table 1 for Japan in near future. Such rising trend in public debt has stimulated a wide-spread concern among the public, policymakers, and researchers about the sustainability of current US fiscal system going forward. This in fact is a large-picture background of the balanced-budget debate and oftentimes a trigger of the debate.

can be a powerful stabilization device to help preempt sunspots expectations and guard the economy against welfare-reducing extrinsic instability.

4.4 Implications for three large economies

To get a more practical feel, in this last step, we apply the calibrated version of our model in Section 4.3 to discuss its implications for the United States, the United Kingdom, and Japan. We find that extrinsic instability arises for capital depreciation allowances and income tax rates typically seen in these large economies, but that the economies can be stabilized if depreciation allowances are sufficiently higher or income tax rates lower than the current levels.

We assign the same values to those deep parameters as they were used in the previous sections, or, $\rho = 0.04$, $\delta = 0.1$, and $\alpha = 0.3$. For income tax rates in the US, UK, and Japan, we use these countries' effective tax rates on factor incomes in 1996 updated from the estimates by Mendoza et al. (1994). The public debt to GDP ratios in the three economies are computed using data from OECD Economic Outlook (2014), where public debt is measured by net financial liabilities held by the public. These income tax rates and public debt-GDP ratios are reported in Table 1.

While tax codes in countries around the world generally feature some degrees of capital depreciation allowances, studies that estimate such a degree are rare. D'Erasmo et al. (2017) contains such a study. These authors observe that, in practice, depreciation allowances typically apply only to the capital of businesses and self-employed, but not to residential capital. Accordingly, they find that for the fifteen largest European countries the GDP-weighted average capital depreciation allowance rate is about 0.2, and that the rate for the United States is also close to 0.2. We therefore take 0.2 as the empirically plausible value of μ for the three large economies considered here.

Key results from our numerical analyses of the calibrated model are summarized in Table 2. As is clear from the table, all of the three economies are extrinsically instable. At the calibrated income tax rates $\tau = (\tau^k, \tau^l)$ reported in Table 1, raising the capital depreciation allowance rate μ from the calibrated level, 0.2, to 0.76, would stabilize everyone. At the calibrated capital depreciation allowance rate $\mu = 0.2$, saddle-path stability would be achieved for an economy if its income tax rates $\tau = (\tau^k, \tau^l)$ are reduced, by 47% for the US, by 52% for the UK, and by 39% for Japan, respectively, from their calibrated levels reported in Table 1. A further tax cut, by an additional 7% for the US, by an additional 6% for the UK, and by an additional 8% for Japan, respectively, would render the respective economy saddle-path stable even if the economy were not to give any tax allowance for capital depreciation (i.e., even if $\mu = 0$).

In all of these counter-factual experiments, the public debt-GDP ratios for the three economies are kept at their calibrated values reported in Table 1. When we consider higher public debt to GDP ratios, increase in capital depreciation allowances or reduction in income tax rates required for preempting extrinsic instability can be more dramatic.

In light of this, some combinations of tax cut and allowance hike which are each more moderate in size can be worth considering. Here are a few examples of fiscal reforms that may help stabilize the economies at their current public debt-GDP ratios:

- For the United States, a 10% cut in income tax rates coupled with a hike in capital depreciation allowance rate to 0.67; Or, a 5% cut in income tax rates combined by a hike in capital depreciation allowance rate to 0.7.

- For the United Kingdom, a 10% cut in income tax rates coupled with a hike in capital depreciation allowance rate to 0.71; Or, a 5% cut in income tax rates combined by a hike in capital depreciation allowance rate to 0.73.

- For Japan, a 10% cut in income tax rates coupled with a hike in capital depreciation allowance rate to 0.59; Or, a 5% cut in income tax rates combined by a hike in capital depreciation allowance rate to 0.62.

This is by no means an exhaustive list but it gives an indication of many possible combinations of tax cut and allowance hike that may work. Nor is this an optimal policy recommendation intended for real world implementation. These calculations are too simplified to capture whole magnitudes or to complete the evaluation of policies including their entire effects. The main intention of this paper is to show some key issues that are either overlooked or misperceived in the existing literature, to point out potential directions in solving the problems, and to see the core mechanisms that produce these negative and positive results, in the most parsimonious and transparent model. We believe that well-thought-out and well-conceived fiscal reforms must take these effects into account.

5 Concluding remarks

The impact of capital income taxation on efficiency or equality has attracted an increasing attention in tax reform and public policy debates. This paper points out some key issues concerning capital tax in its effects on aggregate stability, which are either overlooked or misperceived in the existing literature. We have shown several main results. First, contrary to the conventional wisdom, reliance on adjusting capital tax rate to eliminate short-run fiscal imbalances can create welfare-reducing extrinsic instability. The key to overturning the conventional wisdom is to respect capital utilization as an optimal decision by agents inside the model rather than being fixed by the modeler from outside. Second, also contrary to the common belief, the presence of public debt may aggravate rather than relieve the instability issue. Third, potential resolutions include giving on a permanent basis sufficient tax allowances for capital depreciation expenses or reductions in income tax rates. Combinations of the two measures with more moderate sizes in each can also work. Hence our analyses in this paper also add a short-run motivation to the long-run approach to capital income taxation and the supply-side view of fiscal policy reforms.

This paper also contributes more generally to the broad literature on multiplicity of equilibria and sunspots that is rested upon the preassumption of increasing returns in the aggregate economy. The key critique of this literature is the lack of micro-foundation or empirical ground for the degree of increasing returns to scale needed to be pre-assumed into the aggregate production function. The present paper shows that increasing returns to capital and labor inputs can emerge endogenously from a pre-assumed constant returns aggregate production function due to the interaction between the optimal utilization of capital and the fiscal policy. This mechanism is entirely new and provides a policy-based micro-foundation for aggregate increasing returns. It is this mechanism that renders our model economy prone to extrinsic uncertainty. Potential measures like those demonstrated in this paper can relieve this instability issue exactly because they debilitate this mechanism.

The results in this paper are based on local analyses so the paper is confined to the issue of stationary sunspots fluctuations. It would be worthwhile to conduct a global analysis to investigate the possibility of complicated dynamics related to chaos or limit cycles. Also, this paper follows Schmitt-Grohé and Uribe (1997) and others to study a closed-economy model, and thus the results of this paper may be relevant to large economies like the United States, the United Kingdom, and Japan. The analysis could be extended to a small open economy setting by taking into account cross-border flows in capital and goods. We intend to leave these investigations to future research.

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A Appendix

Proof of Proposition 1.

This is a special case for the proof of Proposition 1 below.

Proof of Corollary 1.

This is a special case for the proof of Corollary 3 below.

Proof of Proposition 2.

In the case with capital depreciation allowance, the fiscal policy is amended as (19) and hence the first-order condition (7) changes to

$$\left(1-\tau_t^k \mu\right) \delta'\left(u_t\right) k_t=\left(1-\tau_t^k\right) r_t k_t, \tag{A.1}$$

the Euler equation (8) becomes

$$\dot{\lambda}_t=\lambda_t\left[\rho+\left(1-\tau_t^k \mu\right) \delta_t-\left(1-\tau_t^k\right) r_t u_t\right]. \tag{A.2}$$

The other features remain the same as the ones in Section 2. Using the linearized versions of (5), (6), (9), (10), (19) and (A.1), we can express y_t, c_t, r_t, u_t and τ_t^k (along with l_t and w_t) in terms of k_t and λ_t. And then we substitute the outcomes into the linearized versions of (12) and (A.2) to obtain (14). The elements of the Jacobian matrix $\mathbf{J}$ are given by,

$$J_{11}=-\frac{(1-\alpha)\left(1-\mu \tau^k\right)\left[\rho+\left(1-\mu \tau^k\right) \delta\right] \epsilon_{11}}{\alpha \rho}, \quad J_{12}=-\tfrac{\lambda}{y} \frac{(\rho+\delta-\mu \delta)\left(1-\mu \tau^k\right)\left[\rho+\left(1-\mu \tau^k\right) \delta\right] \tau^k}{\alpha\left(1-\tau^k\right) \Pi^D},$$

$$J_{21}=\tfrac{y}{\lambda}\left[1-\alpha+\frac{\alpha \rho\left(1-\tau^k\right)}{\rho+\left(1-\mu \tau^k\right) \delta}+\frac{(1-\alpha)\left(1-\tau^k\right)}{\rho} J_{22}\right], \quad J_{22}=\left(1-\mu \tau^k\right) \epsilon_{22},$$

with the four auxiliary notations introduced as follows to help simplify exposition,

$$\epsilon_{11}=\frac{\left\{\left[\rho+\left(1-\mu \tau^k\right) \delta\right](1+v \varepsilon)-(1+\varepsilon) \delta\left(1-\mu \tau^k\right)\right\}\left(1-\tau^k\right)+[\rho+(1-\mu) \delta] \tau^k}{\Pi^D},$$

$$\varepsilon_{22}=\frac{\left[\rho+\left(1-\mu \tau^k\right) \delta\right](1+v \varepsilon)-(1+\varepsilon) \alpha \delta\left(1-\tau^k\right)}{\alpha \Pi^D},$$

$$v=\frac{\mu \delta\left(1-\tau^k\right)}{\rho+\left(1-\mu \tau^k\right) \delta}, \quad \varepsilon=\frac{\rho+\left(1-\mu \tau^k\right) \delta}{\rho+(1-\mu) \delta}\left(\frac{1}{1-\tau^k}-\frac{1}{1-\mu \tau^k}\right).$$

where Π^D, as defined by the left side of (21), is a convex function of τ^k.

The trace and determinant of $\mathbf{J}$ are then obtained as follows,

$$\mathcal{T} = \frac{\overbrace{(1-\mu)\delta\left\{(1-\alpha)\mu\left[\rho+\left(1-\mu\tau^k\right)\delta\right]\left(\tau^k\right)^2+\alpha\rho\left(1-\tau^k\right)\right\}+\alpha\rho\left(1-\mu\tau^k\right)\left[\rho+(1-\mu)\delta\tau^k\right]}^{+}}{\alpha\left[\rho+(1-\mu)\delta\right]\Pi^D}, \tag{A.3}$$

$$\mathcal{D} = \overbrace{\frac{\rho}{\alpha}(1-\alpha)\left(1-\mu\tau^k\right)\left[\rho+\left(1-\mu\tau^k\right)\delta\right]\tau^k}^{+}\frac{\Omega^D}{\Pi^D}, \tag{A.4}$$

where recall that Ω^D is given by the left side of (20), viewed as a function of τ^k. With some algebra, we establish the following properties: as τ^k approaches 0 from the right, Ω^D approaches $-\infty$; as τ^k approaches 1 from the left, Ω^D approaches $+\infty$; and,

$$\Omega^{D\prime}\left(\tau^k\right) = \frac{\rho+(1-\mu)\delta}{\rho\left(1-\tau^k\right)^2}+\frac{\alpha\mu\delta}{1-\alpha}\frac{\rho+(1-\mu)\delta}{\left[\rho+\left(1-\mu\tau^k\right)\delta\right]^2}+\frac{\rho+(1-\alpha)\delta}{\alpha\rho\left(\tau^k\right)^2}+\frac{\delta(\alpha-\mu)\mu}{\alpha(\rho+\delta-\mu\delta)}\frac{1-\mu}{\left(1-\mu\tau^k\right)^2}.$$

It is clear that $\Omega^{D\prime}\left(\tau^k\right)>0$ for $\mu\leq\alpha$ and all τ^k. Because $\Omega^D\left(\tau^k\right)$ is strictly increasing in τ^k, there exists a unique root, denoted as $\bar{\tau}^D\in(0,1)$, satisfying $\Omega^D(\bar{\tau}^D)=0$ such that $\Omega^D(\tau^k)<0$ for all $\tau^k<\bar{\tau}^D$, and $\Omega^D>0$ for all $\tau^k>\bar{\tau}^D$.

For $\mu>\alpha$, the inequality $\Omega^D<0$ is equivalent to

$$\frac{\rho+(1-\mu)\delta}{\left(1-\tau^k\right)\rho}+\frac{\alpha}{1-\alpha}\frac{\rho+(1-\mu)\delta}{\rho+\left(1-\mu\tau^k\right)\delta}<\frac{\rho+(1-\alpha)\delta}{\alpha\rho\tau^k}-\frac{\delta(\mu-\alpha)\mu}{\alpha(\rho+\delta-\mu\delta)}\frac{1-\tau^k}{1-\mu\tau^k}.$$

Denote the left side of the inequality as $LF\left(\tau^k\right)$ and it is easy to verify that $LF'(\tau^k)>0$, $LF(0)=\left[\rho+(1-\mu)\delta\right]s/\left[(1-\alpha)\rho\right]>0$, and $LF(1)=+\infty$. So $LF\left(\tau^k\right)$ is a strictly increasing function of τ^k in the interval of $(0,1)$. Denote the right side of the inequality as $RT\left(\tau^k\right)$, which satisfies $RT(0)=+\infty$, and $RT(1)=\left[\rho+(1-\alpha)\delta\right]/\alpha\rho>0$. Moreover, we can derive that

$$RT'(\tau^k) = \frac{\rho+(1-\alpha)\delta}{\rho\alpha\left(1-\mu\tau^k\right)^2}\left[\zeta^2-\left(\frac{1-\mu\tau^k}{\tau^k}\right)^2\right],$$

where $\zeta^2=\rho\delta\mu(\mu-\alpha)(1-\mu)/\left[(\rho+\delta-\mu\delta)(\rho+\delta-\alpha\delta)\right]$. It is obvious that $RT'\left(\tau^k\right)<(>)\,0$ for all $\tau^k<(>)\,\tau^h$ where $\tau^h=1/(\mu+\zeta)>0$ satisfying $RT'(\tau^k)=0$. So $RT\left(\tau^k\right)$ is strictly decreasing in τ^k in the interval of $(0,\tau^h)$ and strictly increasing in $\left(\tau^h,1\right)$.

If $LF\left(\tau^h\right) > RT\left(\tau^h\right)$, recall that $LF\left(\tau^k\right)$ and $RF\left(\tau^k\right)$ move in the opposite directions for $\tau^k \in (0, \tau^h)$, and also $LF(0) < RT(0)$. There exists a unique root $\bar{\tau}^D \in \left(0, \tau^h\right)$ satisfying $\Omega^D(\bar{\tau}^D) = LF\left(\bar{\tau}^D\right) - RT\left(\bar{\tau}^D\right) = 0$. And we have $\Omega^D(\tau^k) < 0$ for all $\tau^k < \bar{\tau}^D$. However, if $LF\left(\tau^h\right) < RT\left(\tau^h\right)$, recall that $LF\left(\tau^k\right)$ and $RF\left(\tau^k\right)$ move in the same direction for $\tau^k \in (\tau^h, 1)$, and also $LF(1) > RT(1)$. We know that there exists a unique root $\bar{\tau}^D \in \left(\tau^h, 1\right)$ satisfying $\Omega^D(\bar{\tau}^D) = 0$. And we also get $\Omega^D(\tau^k) < 0$ for all $\tau^k < \bar{\tau}^D$.

On the other hand, we can verify that Π^D viewed as a convex function of τ^k has the following properties: $\Pi^D(0) = 1 > 0$, $\Pi^D(1/(1+\omega)) = -\rho\omega/\delta/(1+\omega) \leq 0$ and $\Pi^D(1) = -(\rho/\delta + 1 - \mu)\omega \leq 0$, where the equality holds if $\mu = 1$. These together confirm our claim in the text that $\Pi^D\left(\tau^k\right)$ has a smaller root in $(0, 1/(1+\omega)]$ (denoted by $\underline{\tau}^D$ in the text) and is strictly decreasing in τ^k for all $\tau^k \in \left[0, \underline{\tau}^D\right]$.

Now it is straightforward to show that $\mathcal{T} < 0$ if and only if $\Pi^D < 0$, or $\underline{\tau}^D < \tau^k$. Conditional on $\Pi^D < 0$, then $\mathcal{D} > 0$ if and only if $\Omega^D < 0$, or $\tau^k < \bar{\tau}^D$. This establishes Proposition 2.

Q. E. D.

Proof of Corollary 2.

To simplify exposition, let

$$\underline{\tau}_0 \equiv \frac{\rho^2}{(1+\omega)\left[\rho + (1-\mu)\delta\right]^2}, \quad \underline{\tau}_1 \equiv \frac{\rho}{(1+\omega)\left[\rho + (1-\mu)\delta\right]}.$$

It is easy to verify that $0 < \underline{\tau}_0 < \underline{\tau}_1 < 1$ for all $\mu \in [0, 1]$. With some algebra, we can show that $\Pi^D(\underline{\tau}_1) = \omega/(1+\omega) > 0$. Recall that the convex function $\Pi^D\left(\tau^k\right)$ is strictly decreasing in τ^k for all $\tau^k \in \left[0, \underline{\tau}^D\right]$, where $\underline{\tau}^D$ is the unique solution in $(0, 1)$ to the quadratic equation $\Pi^D\left(\tau^k\right) = 0$. We then must have $\underline{\tau}_1 < \underline{\tau}^D$. Moreover, as $\underline{\tau}^D$ locates on the downward-sloping branch of the convex function Π^D, we must have $d\Pi^D\left(\tau^k\right)/d\tau^k|_{\tau^k = \underline{\tau}^D} < 0$.

Applying the implicit function theorem to the quadratic equation $\Pi^D\left(\underline{\tau}^D\right) = 0$, we can prove that $d\underline{\tau}^D/d\mu > 0$ for all $\mu \in [0,1]$ because

$$\frac{d\underline{\tau}^D}{d\mu} = -\frac{(1+\omega)\overbrace{\left(\underline{\tau}^D - \underline{\tau}_0\right)}^{+} + \frac{2(1-\mu)\delta\rho+[(1-\mu)\delta]^2}{\rho[\rho+(1-\mu)\delta]^2}\delta\left(1-\mu\underline{\tau}^D\right)}{\underbrace{d\Pi^D\left(\tau^k\right)/d\tau^k|_{\tau^k=\underline{\tau}^D}}_{-}}\underline{\tau}^D.$$

With full allowance, i.e. $\mu = 1$, it is straightforward to show that $\Pi^D\left(\tau^k\right) = (1-\tau^k)^2 > 0$ and hence $\mathcal{T} > 0$ according to (A.3). Hence, there cannot exist two negative eigenvalues of $\mathbf{J}$ and thus indeterminacy can never occur. This establishes Corollary 2.

Q. E. D.

Proof of Corollary 3.

After some rearrangement, we can rewrite the determinant of $\mathbf{J}$ in the proof of Proposition 2, (A.4), as

$$\mathcal{D} = \frac{\rho}{\alpha}(1-\alpha)\frac{\rho+\left(1-\mu\tau^k\right)\delta}{1-\tau^k}\left(\frac{1}{\widetilde{\beta}^D - 1} - \frac{1}{\xi}\right),$$

with the two auxiliary notations introduced as follows to help simplify exposition,

$$\xi = -\frac{\Pi^D}{\Gamma^D},$$

$$\Gamma^D = \left[\frac{1}{1-\tau^k} + \frac{(1-\mu)\delta\mu\tau^k}{(\rho+\delta-\mu\delta)\left(1-\mu\tau^k\right)} + \frac{\alpha}{1-\alpha}\frac{\rho+\delta-\mu\delta}{\rho+\left(1-\mu\tau^k\right)\delta}\right]\left(1-\tau^k\right)\left(1-\mu\tau^k\right)\tau^k > 0.$$

As our claim in the text, indeterminacy occurs if and only if $\Pi^D < 0$ (i.e. $\mathcal{T} < 0$) and $\mathcal{D} > 0$, which implies three necessary conditions for indeterminacy as:

Condition 1. $\widetilde{\beta}^D > 0$.

Recall that $\widetilde{\beta}^D = \left(1+\eta_f^D\right)\eta_l^D\beta$. Because η_l^D is positive, Condition 1 means that $1+\eta_f^D > 0$. With some algebra, we get

$$1+\eta_f^D = \frac{s^D\left(1-\tau^k\right)}{s^D - \left(1+s^D-s\right)\tau^k}.$$

Note that $s^D > 0$ for $\mu \in [0,1)$. When indeterminacy occurs, we must have $s^D - \left(1+s^D-s\right)\tau^k > 0$ and hence $\eta_f^D > 0$. That is to say that the effective degree of returns to scale of the aggregate production function must be larger than 1, i.e. $1+\eta_f^D > 1$.

Condition 2. $\widetilde{\beta}^D - 1 > 0$.

This condition implies that the effective labor demand curve is upward sloping and steeper than the labor supply curve.

Condition 3. $\widetilde{\beta}^D < 1 + \xi$.

From this inequality condition, we know that the *fiscal* increasing returns to scale cannot be too large.

Combining Conditions 2 and 3, the necessary and sufficient condition for indeterminacy in Proposition 2 can be rewritten as

$$1 < \widetilde{\beta}^D < 1 + \xi.$$

This condition is similar as the one in the model with factor-generated externalities. This establishes Corollary 3.

Q. E. D.

Table 1. Estimated effective income tax rates[a] and public debt-GDP ratio[b]

	τ^k	s_b	τ^l
US	0.3962	0.855	0.27733
UK	0.4717	0.642	0.24406
JP	0.4261	1.429	0.27439

[a] Effective tax rates on factor incomes in 1996 updated from the estimates by Mendoza et al. (1994), available at http://www.sas.upenn.edu/~egme/pp/newtaxdata.pdf.
[b] Public debt-GDP ratios computed using data from OECD Economic Outlook (2014). Public debt is measured by net financial liabilities held by the public.

Table 2. (In)stability properties for different countries

	US			UK			JP		
μ \ τ	τ	0.53τ	0.46τ	τ	0.48τ	0.42τ	τ	0.61τ	0.53τ
0	I	I	S	I	I	S	I	I	S
0.2*	I	S	S	I	S	S	I	S	S
0.76	S	S	S	S	S	S	S	S	S

Notes: $\tau = (\tau^k, \tau^l)$ – calibrated capital income tax rate τ^k and labor income tax rate τ^l as reported in Table 1. * indicates 0.2 as the empirically relevant capital depreciation allowance rate. I – instability; S – stability.

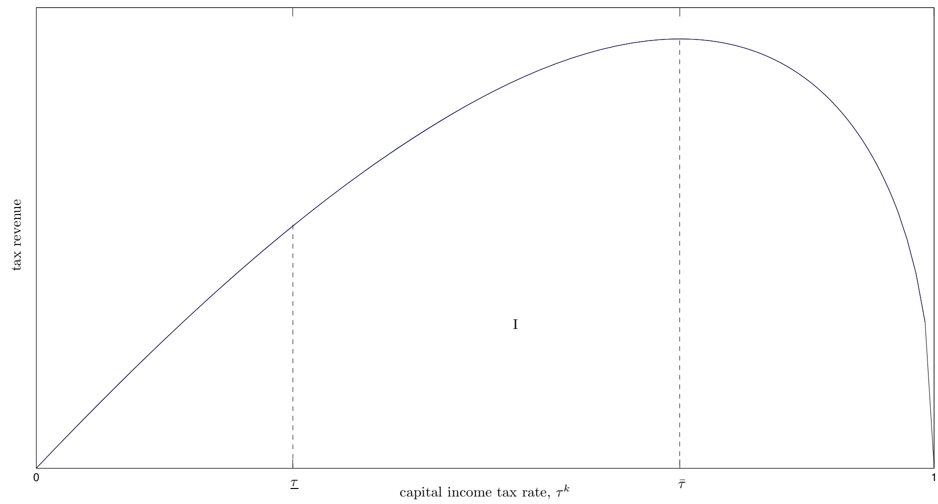

Figure 1. Steady-state Laffer curve. I - instability.

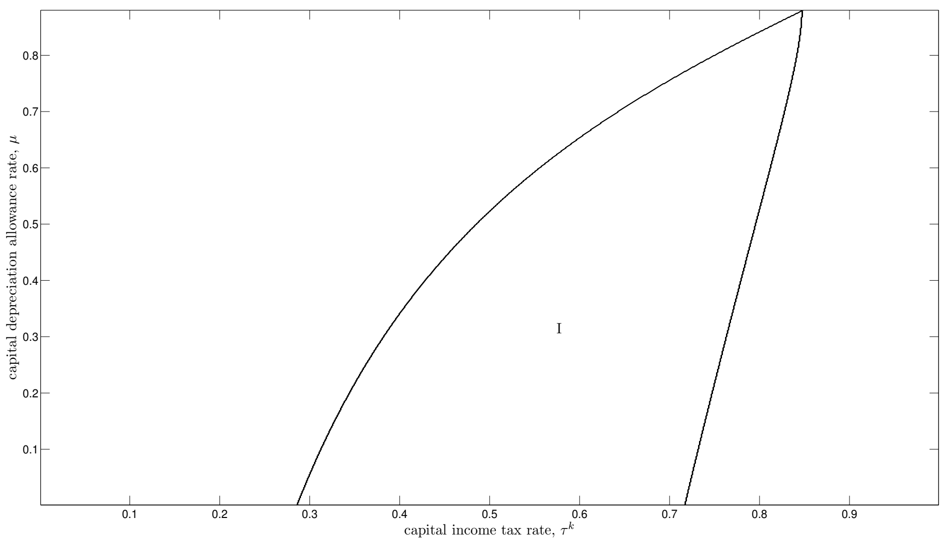

Figure 2. Capital income taxation with depreciation allowances. I - instability.

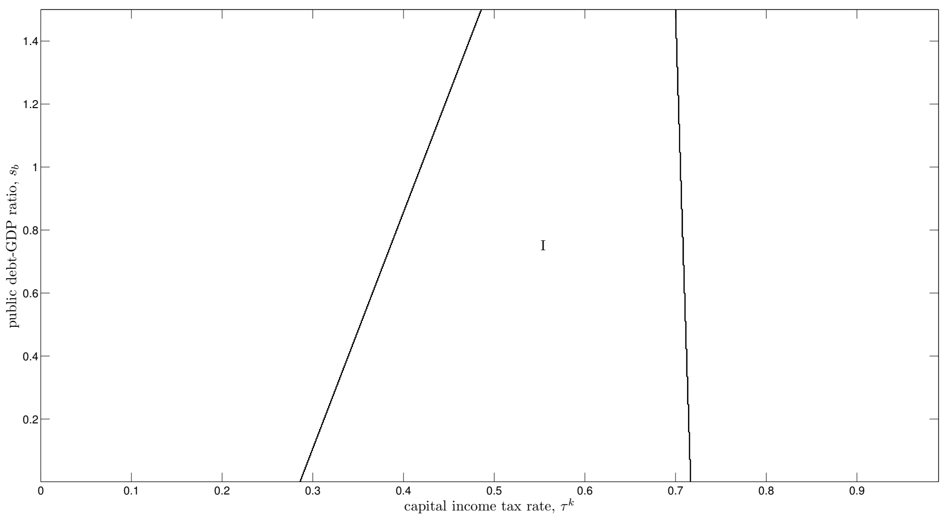

Figure 3. Capital income taxation with public debt. I - instability.

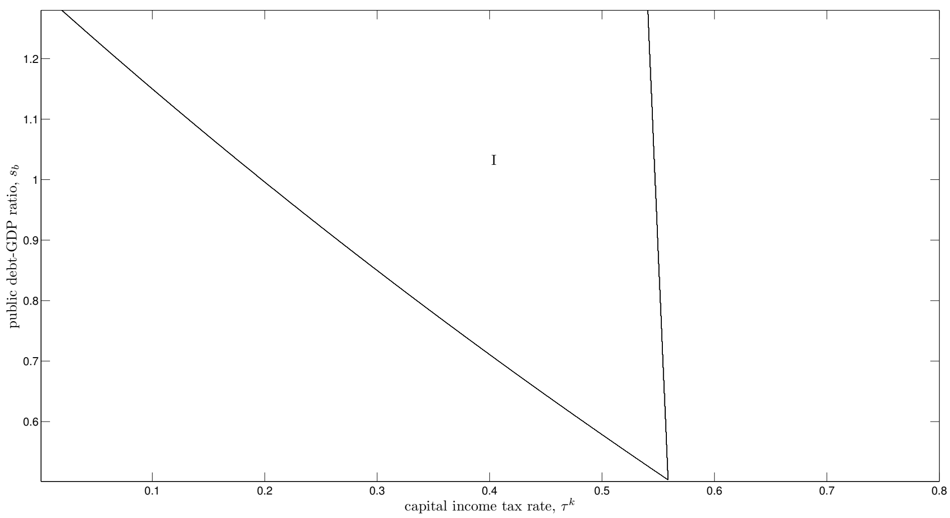

Figure 4. Capital income taxation with public debt and fixed labor income tax rate. I - instability.

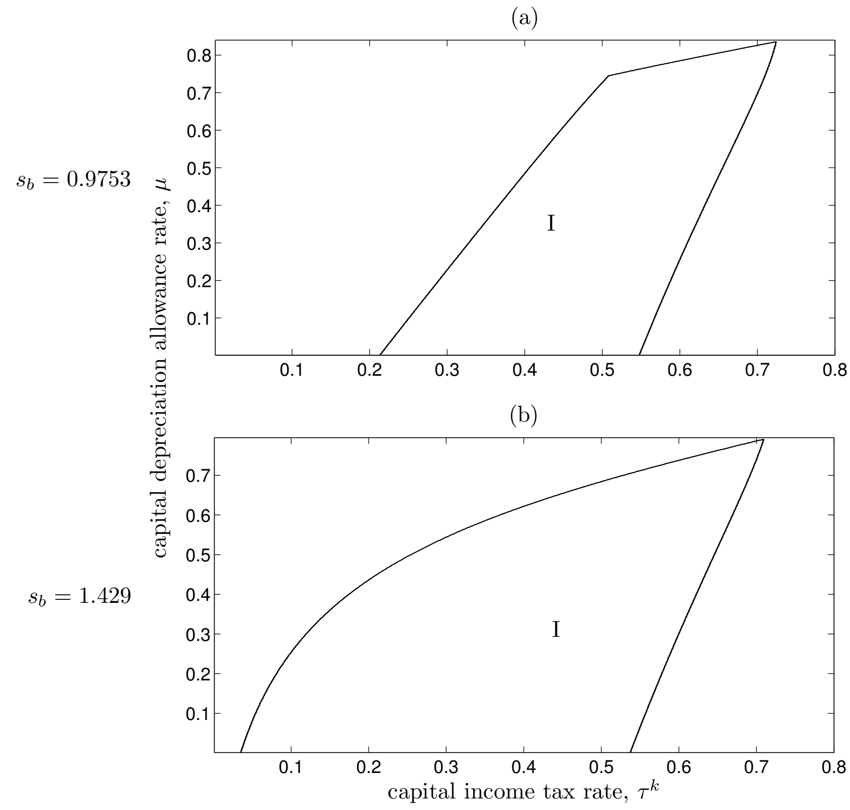

Figure 5. Capital income taxation with depreciation allowances, and fixed labor income tax rate and public debt. I - instability.